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Extinction properties of solutions for a class of fast diffusive p-Laplacian equations

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A B S T R A C T

We consider the extinction properties of solutions for the homogeneous Dirichlet boundary value problem for the p-Laplacian equation $u_t - \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) + \beta u^q = \lambda u^r$ with $1 < p < 2$, $q \leq 1$ and $r, \lambda, \beta > 0$. For $\beta = 0$, it is known that $r = p - 1$ is the critical extinction exponent for the weak solution. For $\beta > 0$, we show that $r = p - 1$ is still the critical extinction exponent when $q = 1$. Moreover, the precise decay estimates of solutions before the occurrence of the extinction are derived. However, extinction can always occur when $0 < q \leq r < 1$.

1. Introduction and the main results

This paper deals with the extinction properties of solutions for the p-Laplacian equation

$$u_t - \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) + \beta u^q = \lambda u^r, \quad x \in \Omega,\ t > 0, \tag{1.1}$$

subject to the initial and boundary value conditions

$$u(x,t) = 0, \quad x \in \partial\Omega,\ t > 0, \tag{1.2}$$

$$u(x,0) = u_0(x), \quad x \in \Omega, \tag{1.3}$$

where $1 < p < 2, q \leq 1, r, \lambda, \beta > 0, \Omega \subset \mathbb{R}^N$ $(N > 2)$ is an bounded domain with smooth boundary and $u_0(x) \in L^\infty(\Omega) \cap W_0^{1,p}(\Omega)$ is a nonzero non-negative function.

Eq. (1.1) is a class of nonlinear singular parabolic equations and appears to be relevant in the theory of non-Newtonian fluids perturbed by both a nonlinear reaction term and an absorption term; see [1–4] for instance. It is also relevant in combustion theory, where the function $u(x,t)$ represents the temperature, the term $-\operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right)$ represents the thermal diffusion, βu^q represents the absorption and λu^r is a source.

Extinction is the phenomenon whereby the evolution of some nontrivial initial data $u_0(x)$ produces a nontrivial solution $u(x,t)$ in a time interval $0 < t < T$ and then $u(x,t) \equiv 0$ for all $(x,t) \in \Omega \times [T, +\infty)$. It is an important property of solutions for many evolution equations which have been studied extensively by many researchers (see [5–10]). In particular, there are also some papers concerning extinction for the problem (1.1)–(1.3) for special cases. For instance, Dibenedetto [2] and Yuan et al. [11] proved that the necessary and sufficient condition for the extinction to occur is $p \in (1,2)$ for the case $\beta = \lambda = 0$.

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