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Cauchy matrix for linear almost periodic systems and some consequences

Manuel Pinto, Gonzalo Robledo *

Departamento de Matemáticas, Facultad de Ciencias, Universidad de Chile, Casilla 653, Santiago, Chile

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ABSTRACT

A novel method to construct the fundamental matrix for a linear almost periodic system is proposed, provided that the diagonal terms satisfy an average separation condition and the off-diagonal coefficients are L^∞-small. The idea is to transform the system in a set of Riccati type equations and use exponential dichotomy and its consequences. It is shown that the method yields easy computation procedures with simple and direct conditions depending on the coefficients. Finally, our result enables us to obtain: (i) explicit almost periodic matrices $Q(t)$, $Q^{-1}(t)$ and $Q'(t)$, which diagonalize the original system and (ii) sufficient conditions for the stability. Two illustrative examples are shown.

1. Introduction

The purpose of this work is to build a fundamental matrix $X(t)$ for the system

$$x' = A(t)x, \tag{1.1}$$

where $x = \mathrm{col}(x_1, \ldots, x_n) \in \mathbb{C}^n$ and $A(t)$ is a matrix with (Bohr) almost periodic coefficients $a_{ij}: \mathbb{R} \mapsto \mathbb{C}$.

Almost periodic functions are an attractive topic in qualitative theory of ordinary and partial differential equations (see *e.g.*, [1–4]) due to their applications in engineering [5–8], life sciences [9–11], and other topics. We assume that the reader is familiar with almost periodic functions and exponential dichotomy theory: we used [12,1] and [13] as general references for the classical theory. For any almost periodic function $f(\cdot)$, its mean value will be defined as follows:

$$\mathcal{M}\{f\} = \lim_{T\to+\infty} \frac{1}{T}\int_0^T f(s)\,ds. \tag{1.2}$$

The connection between exponential dichotomy, integral separation and diagonalizability of (1.1) is a classic topic in qualitative theory of differential equations. The computation of a fundamental matrix is a key matter and its difficulty triggered the study of asymptotic integration theory.

1.1. Some results concerning (1.1)

We will present some results about system (1.1), some of them are associated with the almost periodic framework and other ones are presented in a nonautonomous, bounded and continuous context.

* Corresponding author. Tel.: +56 55 2 9787198.
E-mail addresses: pintoj@uchile.cl (M. Pinto), grobledo@uchile.cl, robledoveloso@gmail.com (G. Robledo).

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