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A Neumann problem for the KdV equation with Landau damping on a half-line

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ABSTRACT

We consider the initial-boundary value problem on a half-line for the KdV equation with Landau damping. We study traditionally important problems of the theory of nonlinear partial differential equations, such as global in time existence of solutions to the initial-boundary value problem and the asymptotic behavior of solutions for large time.

1. Introduction

We consider the initial-boundary value problem on a half-line for the KdV equation with Landau damping:

$$\begin{cases} u_t + uu_x - |\alpha| u_{xxx} + \mathcal{R}^{\frac{1}{2}} \partial_x u = 0, \quad t > 0,\ x > 0, \\ u(x,0) = u_0(x), \quad x > 0, \\ u_x(0,t) = u_{xx}(0,t) = 0, \quad t > 0, \end{cases} \tag{1.1}$$

where $\alpha \in \mathbb{R}$, and

$$\mathcal{R}^{\beta}\phi = \frac{1}{2\Gamma(\beta)\sin\left(\frac{\pi}{2}\beta\right)} \int_0^{+\infty} \frac{\phi(y)}{|x-y|^{1-\beta}} dy$$

is the modified Riesz potential (see [1], p. 214).

Due to the intensive development of the theory and applications, the initial-boundary value problem (1.1) plays an important role in modern science. Apart from diverse areas of mathematics, nonlocal partial differential evolution equations arise in modern mathematical physics and many other branches of science, such as, for example, chemical physics and electrical networks (for details, see [2–9]).

Many articles have appeared in the literature, where fractional derivatives are used for a better description of certain material properties. For example, Ott et al. [10–12] proposed the following generalizations of the KdV equation

$$u_t + uu_x + \alpha u_{xxx} + \mathcal{R}^{\frac{1}{2}} \partial_x u = 0. \tag{1.2}$$

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