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Interior $C^{0,\gamma}$-regularity for vector-valued minimizers of quasilinear functionals

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ABSTRACT

We consider weak minimizers of variational integrals whose integrands are quasilinear. Under suitable conditions on the integrand, we obtain results on interior everywhere Hőlder regularity for the weak minimizers. More precise results on the partial regularity can be deduced from our main result too. Some illustrating examples are given at the end of the paper.

1. Introduction

In this paper, we give conditions guaranteeing that the functions minimizing variational integrals

$$\mathcal{A}(u;\Omega)=\int_\Omega A_{ij}^{\alpha\beta}(u)D_\alpha u^i D_\beta u^j\,\mathrm{d}x \tag{1}$$

belong to $C^{0,\gamma}(\Omega,\mathbb{R}^N)$. Here $u:\Omega\to\mathbb{R}^N$, $N>1$, $\Omega\subset\mathbb{R}^n$, $n\geq 3$ is a bounded open set, $x=(x_1,\dots,x_n)\in\Omega$, $u(x)=(u^1(x),\dots,u^N(x))$, $Du=\{D_\alpha u^i\}$, $D_\alpha=\partial/\partial x_\alpha$, $\alpha=1,\dots,n$, $i=1,\dots,N$. Throughout the whole text, we use the summation convention over repeated indices. We call a function $u\in W^{1,2}(\Omega,\mathbb{R}^N)$ a minimizer of the functional $\mathcal{A}(u;\Omega)$ if and only if $\mathcal{A}(u;\Omega)\leq\mathcal{A}(v;\Omega)$ for every $v\in W^{1,2}(\Omega,\mathbb{R}^N)$ with $u-v\in W_0^{1,2}(\Omega,\mathbb{R}^N)$. For more information, see [1,2].

On the functional $\mathcal{A}$, we assume the following conditions.

(i) $A_{ij}^{\alpha\beta}=A_{ji}^{\beta\alpha}$, $A_{ij}^{\alpha\beta}$ are continuous functions in $u\in\mathbb{R}^N$ and there exists $M>0$ such that $|A_{ij}^{\alpha\beta}(u)|\leq M$, $\forall\,u\in\mathbb{R}^N$.

(ii) Ellipticity: there exists $\nu>0$ such that

$$A_{ij}^{\alpha\beta}(u)\xi_\alpha^i\xi_\beta^j\geq\nu|\xi|^2,\quad\forall\,u\in\mathbb{R}^N,\ \forall\,\xi\in\mathbb{R}^{nN}. \tag{2}$$

(iii) Oscillation of coefficients: there exists a real function ω continuous on $[0,\infty)$, which is bounded, nondecreasing, concave, $\omega(0)=0$ and such that for all $u,v\in\mathbb{R}^N$

$$\left|A_{ij}^{\alpha\beta}(u)-A_{ij}^{\alpha\beta}(v)\right|\leq\omega\left(|u-v|\right). \tag{3}$$

We set $\omega_\infty=\lim_{t\to\infty}\omega(t)\leq 2M$.

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