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Large time existence for 1D Green-Naghdi equations

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ABSTRACT

We consider here the 1*D* Green-Naghdi equations that are commonly used in coastal oceanography to describe the propagation of large amplitude surface waves. We show that the solution of the Green-Naghdi equations can be constructed by a standard Picard iterative scheme so that there is no loss of regularity of the solution with respect to the initial condition.

1. Introduction

1.1. Presentation of the problem

The water-waves problem for an ideal liquid consists of describing the motion of the free surface and the evolution of the velocity field of a layer of perfect, incompressible, irrotational fluid under the influence of gravity. This motion is described by the free surface Euler equations that are known to be well-posed after the works of Nalimov [1], Yosihara [2], Craig [3], Wu [4,5] and Lannes [6]. But, because of the complexity of these equations, they are often replaced for practical purposes by approximate asymptotic systems. The most prominent examples are the Green-Naghdi equations (GN) – which is a widely used model in coastal oceanography ([7–9] and, for instance, [10,11]) –, the Shallow-Water equations, and the Boussinesq systems; their range of validity depends on the physical characteristics of the flow under consideration. In other words, they depend on certain assumptions made on the dimensionless parameters ε, μ defined as:

$$\varepsilon = \frac{a}{h_0}, \qquad \mu = \frac{h_0^2}{\lambda^2};$$

where a is the order of amplitude of the waves and the bottom variations; λ is the wavelength of the waves and the bottom variations; h_0 is the reference depth. The parameter ε is often called the nonlinearity parameter; while μ is the shallowness parameter. In the shallow-water scaling ($\mu \ll 1$), and without a smallness assumption on ε one can derive the so-called Green-Naghdi equations (see [7,12] for a derivation and [13] for a rigorous justification) also called Serre or fully nonlinear Boussinesq equations [14].

In nondimensionalized variables, denoting by $\zeta(t, x)$ and $u(t, x)$ the parameterization of the surface and the vertically averaged horizontal component of the velocity at time t, and by $b(x)$ the parameterization of the bottom, the equations read

$$\begin{cases} \partial_t \zeta + \nabla \cdot (hu) = 0, \\ (h + \mu h \mathcal{T}[h, \varepsilon b])\partial_t u + h\nabla\zeta + \varepsilon h(u\cdot\nabla)u + \mu\varepsilon \left\{ -\frac{1}{3}\nabla[h^3((u\cdot\nabla)(\nabla\cdot u) - (\nabla\cdot u)^2)] + h\Re[h, \varepsilon b]u \right\} = 0, \end{cases} \tag{1}$$

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