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Decay estimates for homogeneous Boussinesq equations in a semi-infinite pipe☆

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ABSTRACT

In this paper, we establish the spatial decay bounds for homogeneous Boussinesq equations in a semi-infinite pipe flow. Assuming that the entrance velocity and magnetic field data are restricted appropriately, and it converges to laminar flow as the distance down the pipe tends to infinity, we derive a second order differential inequality that leads to an exponential decay estimate for the energy $E(z,t)$ defined in (27). We also indicate how to establish the explicit bound for the total energy.

1. Introduction

A number of papers in the literature have dealt with the transient flow of an incompressible viscous fluid in a semi-infinite pipe or channel. If the net flow into the finite end of the pipe or channel at any time is not zero, then the flow velocity cannot go to zero as the distance from the finite end tends to infinity. The result can be seen as a version of the Saint-Venant Principle. Early work on Saint-Venant's principle primarily focused on the initial-boundary value problems involving elliptic equations (see [1–4]). It is Boley [5] who, in the 1950s, first pointed out the validity of Saint-Venant's principle for the heat equations. Since then, extensive attention has been paid to the parabolic problems, (see [6–11]). These studies are motivated by a desire to establish,for parabolic equation, spatial decay estimates analogous to those obtained for elliptic equations in the investigation of Saint-Venant's principle in the framework of applied mathematics and mechanics. For a review of recent advance on Saint-Venant's principle, one may refer to [12–14] and the references cited therein.

Making use of explicit upper bounds for solutions of the transient heat conduction equation in a half space, Boley [5] was the first to assert that the spatial influence of the transient effects was even more localized than that of steady state. Then Boley [15] considered the more traditionary initial-boundary value problem for cylindrical domains or semi-infinite strips subject to non-zero boundary conditions on the ends only and obtained some illustrative results, which showed, for

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