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Nonexistence of the reversed flow solutions of the Falkner–Skan equations

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ABSTRACT

Nonexistence of reversed flow solutions of the well-known Falkner–Skan equations arising in the boundary layer theory is considered analytically. A new system of two singular integral equations are proposed and studied, which plays a key role in the study of reversed flow solutions. The properties of the velocity and the shear stress of the reversed flows are provided. These properties describe the shapes and behaviors of the curves of the velocity and the shear stress functions. A new lower bound of the skin-friction which is useful in numerical analysis is given. The results on the nonexistence of reversed flow solutions can be used to estimate the exact critical value which is of importance in aeronautics because separation occurs at this value.

1. Introduction

Boundary layer equations governing the flow of fluids over surfaces can be found commonly in engineering problems. Using a similar transformation, the governing partial differential equation derived from the Navier–Stokes equations can be changed into the well-known Falkner–Skan equation with the usual boundary conditions

$$\begin{cases} f'''(\eta)+f(\eta)f''(\eta)+\lambda[1-(f')^2(\eta)]=0 \quad \text{on } \eta\in(0,\infty), \\ f(0)=f'(0)=0, \quad f'(\infty)=1. \end{cases} \tag{1.1}$$

We refer to Section 7.2 in [1], Sections 9.4–9.5 in [2] and Section 10.3.2 in [3] for the derivation.

One of the class of solutions which is of greatest interest from the point of view of boundary layer theory is the class of reversed flow solutions of (1.1), that is, the solutions f satisfying $f'(\eta) < 0$ for small positive values of η and $f''(0) < 0$. There has been extensive study on the reversed flow solutions of (1.1) via numerical methods, for example see [4–7]. It is proved numerically in [7] that (1.1) has at least one reversed flow solution for each $\lambda \in (\lambda_0, 0)$ and has no reversed flow solutions for $\lambda < \lambda_0$, where $\lambda_0 = -0.1988$. Such a value λ_0 is called a critical value. The numerical solutions were calculated by assuming values of the skin-friction $f''(0)$.

The critical value λ_0 is of importance in aeronautics because separation occurs at λ_0, which would cause serious consequences in aeronautics. For example, separation of the boundary layer can destroy the bound vortex on lifting bodies such as airfoils; see Section 9.11 of Currie [2] and Stewartson [7].

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