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The Calderón–Zygmund property for quasilinear divergence form equations over Reifenberg flat domains

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ABSTRACT

The results by Palagachev (2009) [3] regarding global Hölder continuity for the weak solutions to quasilinear divergence form elliptic equations are generalized to the case of nonlinear terms with optimal growths with respect to the unknown function and its gradient. Moreover, the principal coefficients are discontinuous with discontinuity measured in terms of small *BMO* norms and the underlying domain is supposed to have fractal boundary satisfying a condition of Reifenberg flatness. The results are extended to the case of parabolic operators as well.

1. Introduction

The general aim of the paper is to study regularity issues regarding the Dirichlet problem

$$\begin{cases} \operatorname{div}\left(a^{ij}(x,u)D_ju + a^i(x,u)\right) = b(x,u,Du) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{1.1}$$

over a *Reifenberg flat* and bounded domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$. The nonlinear ingredients are of Carathéodory type (that is, measurable with respect to $x \in \Omega$ for all $(z,\xi) \in \mathbb{R}\times\mathbb{R}^n$ and continuous with respect to $(z,\xi) \in \mathbb{R}\times\mathbb{R}^n$ for almost all (a.a.) $x \in \Omega$), and the principal coefficients $a^{ij}(x,\cdot)$ are allowed to be discontinuous with respect to $x \in \Omega$ with discontinuity measured in terms of appropriate smallness of their *bounded mean oscillation* (*BMO*). Regarding the lower-order terms, we assume that these satisfy *optimal growth conditions* with respect to u and Du, which are the sufficient ones to guarantee the relevant definition of weak solution to the problem (1.1). Precisely, we suppose $a^i(x,z) = \mathcal{O}\left(\varphi_1(x) + |z|^{\frac{n}{n-2}}\right)$ and $b(x,z,\xi) = \mathcal{O}\left(\varphi_2(x) + |z|^{\frac{n+2}{n-2}} + |\xi|^{\frac{n+2}{n}}\right)$ as $|z|, |\xi| \to \infty$ with $\varphi_1 \in L^p(\Omega)$, $p > n$ and $\varphi_2 \in L^q(\Omega)$, $q > n/2$.

Our main result asserts that the problem (1.1) supports the Calderón–Zygmund property. Namely, each $W_0^{1,2}(\Omega)$-weak solution to (1.1) gains better regularity from the data φ_1 and φ_2, and belongs to the Sobolev space $W^{1,\min\{p,q^*\}}(\Omega)$ where q^*

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