Nonlinear Analysis 74 (2011) 3046–3054

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Controllable Hopf bifurcations of codimension 1 and 2 in nonlinear control systems

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ARTICLE INFO

Article history:
Received 2 April 2010
Accepted 25 January 2011

MSC:
37G15
34D20
47N70

Keywords:
Hopf bifurcation
Bautin bifurcation
Stability
Periodic orbit
Control system

ABSTRACT

The purpose of this work, given a nonlinear control system, is to design a four-parameter family of static state feedback such that the corresponding closed-loop control system exhibits controllable Hopf bifurcations of codimension 1 and 2. More precisely, the scalar law designed by us permits the control of the stability of the equilibrium points and the orientation and stability of the periodic orbits.

1. Introduction and statement of the problem

In recent years there has been considerable interest in the so called control of bifurcations in dynamical systems. The studies found in the literature mainly deal with the control of bifurcations of codimension 1. For example, the control of codimension 1 Hopf bifurcations was studied in [1,2]. Relatively little is known about the control of codimension 2 bifurcations from the analytical point of view. As far as we know the study of the control of codimension 2 bifurcations is restricted to the cases of the Bogdanov–Takens bifurcation [3] and Bautin bifurcation [4], both for planar systems.

The main goal of this paper is to develop a control method for nonlinear systems in any dimension that enables the creation of Hopf bifurcations with special interest in degenerate ones. More precisely, the purpose of this paper is, from a nonlinear system, to design a class of four-parameter families of differential equations that exhibits *controllable* Hopf bifurcations of codimension 1 and 2, that is Hopf bifurcations that present all the possible pertinent bifurcation diagrams.

Consider the following nonlinear control system:

$$x' = F(x) + G(x)u, \qquad (1)$$

where $F, G : \mathbb{R}^n \to \mathbb{R}^n$ are smooth vector fields, $x \in \mathbb{R}^n$ is the state vector and $u \in \mathbb{R}$ is the control input.

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0362-546X/$ – see front matter
doi:10.1016/j.na.2011.01.033