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Global existence of solutions to a parabolic–parabolic system for chemotaxis with weak degradation

E. Nakaguchi [a], K. Osaki [b,*]

[a] College of Liberal Arts and Sciences, Tokyo Medical and Dental University, 2-8-30 Kohnodai, Ichikawa, Chiba 272-0827, Japan

[b] Department of Mathematical Sciences, School of Science and Technology, Kwansei Gakuin University, 2-1 Gakuen, Sanda 669-1337, Japan

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ABSTRACT

We study the global existence of solutions to a parabolic–parabolic system for chemotaxis with a logistic source in a two-dimensional domain, where the degradation order of the logistic source is weaker than quadratic. We introduce nonlinear production of a chemoattractant, and show the global existence of solutions under certain relations between the degradation and production orders.

1. Introduction

Chemotaxis is the movement of biological individuals towards (or away from) a chemoattractant (or chemorepellent). It plays a crucial role in the study of chemotactic phenomena such as wound healing, cancer growth and leukocyte movement [1,2]. Bacterial species such as *E. coli* exhibit chemotaxis with respect to the chemoattractant aspartate which they themselves produce. Budrene and Berg [3,4] found that *E. coli* formed remarkable macroscopic regular patterns in their colony resulting from the interplay between chemotaxis, proliferation and the reduction in numbers due to death (we refer to the proliferation and reduction in numbers together simply as growth). In fact, growth over time is necessary for the colony patterns to be developed; sufficient time is required for several generations of *E. coli* to proliferate and die, thereby generating the pattern formation processes.

Many mathematical models have been proposed to elucidate the mechanisms behind these patterns generated by bacterial species. Several of them are based on partial differential equations including terms for chemotaxis and growth. In the equations, chemotaxis can be expressed as a *negative diffusion* that is a directed flux toward the gradient of chemical concentration, and growth as a saturating logistic function. Mimura and Tsujikawa [5] proposed a chemotaxis-growth system of equations, and a simplified version is of the form:

* Corresponding author. Tel.: +81 79 565 7109; fax: +81 79 565 9077.
E-mail addresses: nakaguti.las@tmd.ac.jp (E. Nakaguchi), osaki@kwansei.ac.jp (K. Osaki).

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