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Common coupled fixed point results for probabilistic φ -contractions in Menger spaces*

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ABSTRACT

Several hybrid probabilistic contractions with a gauge function φ are considered. Using the properties of the pseudo-metric and the triangular norm, some common coupled fixed point theorems are obtained in complete Menger probabilistic metric spaces. The main results are general because they do not assume any continuity or monotonicity conditions for φ .

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1. Introduction and preliminaries

Coupled fixed points and their applications for binary mappings were considered by Bhaskar and Lakshmikantham [1]. More explicitly, let *X* be a non-empty set and $T : X \times X \rightarrow X$ be a mapping; then an element $(u, v) \in X \times X$ is called a coupled fixed point of *T* if T(u, v) = u and T(v, u) = v. Recently, some new results for the existence and uniqueness of coupled fixed points were presented for the cases of partially ordered metric spaces, cone metric spaces and fuzzy metric spaces (for examples, see [1–6]).

It is well known that the probabilistic metric space is an important generalization of the metric space and appears to be of interest in the investigation of physical quantities and physiological thresholds (see [7–11]). Fixed point properties for mappings defined on probabilistic metric spaces have been studied by many authors (see [7–16]). In particular in [16], some fixed point theorems for probabilistic nonlinear contractions with a gauge function φ were proved, and relations between several assumptions concerning φ were discussed carefully.

Inspired by the work of [1,2,6,12,16], in this paper we introduce and investigate several hybrid probabilistic contractions with a gauge function φ . In Sections 2 and 3, using the properties of the pseudo-metric and the triangular norm, we prove some common coupled fixed point theorems in Menger probabilistic metric spaces and in non-Archimedean Menger probabilistic metric spaces, respectively. Our results are general because they do not assume any continuity or monotonicity conditions for φ . We finally give some remarks and examples concerning our results in Section 4.

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