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The Cauchy problem for a class of the multidimensional Boussinesq-type equation

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ABSTRACT

In this paper, we study the Cauchy problem for a class of the multidimensional Boussinesq-type equation $u_{tt} - \Delta u + \Delta^2 u + \Delta^2 u_{tt} = \Delta f(u)$, where $f(u) = \pm a|u|^p$ or $-a|u|^{p-1}u$, $a > 0$ is a constant. First, we establish a local existence theorem for the solution. Then, for $m = 1$ $(n = 1)$, $m = 2, 3, 4$ $(n \leq 3)$, we prove the existence of a global H^m solution. Finally, we prove the global nonexistence and finite-time blow up of the solution.

1. Introduction

This paper considers the Cauchy problem for a class of multidimensional Boussinesq-type equation

$$u_{tt} - \Delta u + \Delta^2 u + \Delta^2 u_{tt} = \Delta f(u), \quad x \in \mathbb{R}^n, \ t > 0, \tag{1.1}$$

$$u(x, 0) = u_0(x), \qquad u_t(x, 0) = u_1(x), \quad x \in \mathbb{R}^n, \tag{1.2}$$

where $f(u)$ satisfies one of the following three assumptions:

$$(\mathrm{H})\ f(u) = \pm a|u|^p \quad \text{or} \quad -a|u|^{p-1}u, \quad a > 0, p > 1,$$

$$(\mathrm{H}_1)\ \begin{cases} f(u) = \pm a|u|^p, & a > 0,\ p > 1,\ p \neq 2k,\ k = 1, 2, \ldots \\ \text{or} \quad f(u) = -a|u|^{p-1}u, & a > 0,\ p > 1,\ p \neq 2k+1,\ k = 1, 2, \ldots \end{cases}$$

or

$$(\mathrm{H}_2)\ \begin{cases} f(u) = \pm a u^{2k}, & a > 0,\ k = 1, 2, \ldots \\ \text{or} \quad f(u) = -a u^{2k+1}, & k = 1, 2, \ldots. \end{cases}$$

It is well known that the Boussinesq equation

$$u_{tt} - u_{xx} + (u_{xx} \pm u^2)_{xx} = 0$$

is a very important and famous nonlinear evolution equation which was suggested to describe the motion of water with small amplitude and long wave. There have been many results on the local and global well-posedness of the Cauchy problem for the generalized Boussinesq equation

$$u_{tt} - u_{xx} + (u_{xx} + f(u))_{xx} = 0; \tag{1.3}$$

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