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On the stability of a domain-wall brane model

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ABSTRACT

We establish stability and nonstability results for a domain-wall brane model arising in classical field theory. In particular, we show the nonexistence of nontrivial bounded solutions on the real line for a coupled pair of parameter dependent linear second order ordinary differential equations for an open set of those parameters. Moreover, we establish the existence of nontrivial solutions for a hypersurface of the parameters. We use Fredholm theory for compact linear operators combined with the Lyapunov–Schmidt method to prove our results. The model is stable, respectively unstable, for those parameters for which the coupled system does not, respectively does, have nontrivial solutions.

1. Introduction

In [1] the first author, Thompson, and Volkas build a classical field theory model which describes the observable universe as a $3 + 1$-dimensional brane embedded in a $4 + 1$-dimensional space time manifold. The idea is that the classical theory will be quantized and become part of some new physics which becomes effective at a currently experimentally unprobed high energy scale.

Each $4 + 1$-dimensional field theoretical model may be described by a $4 + 1$-dimensional action written in terms of fields which are identified with particles belonging to a grand unified theory (GUT) [2,3] containing the standard model of particle physics. Brane world models make use of fields belonging to the extended frame work of the grand unified theory, such as GUT scale Higgs fields and solitary waves, to create a brane and localize standard model particles thereon.

Thompson and Volkas identify the most general $4 + 1$-dimensional Minkowski flat space action which is invariant under both global SO(10) transformations and $4 + 1$-dimensional Poincaré transformations for a scalar field Φ and a 45 dimensional adjoint Higgs fields Ψ. For computational purposes the adjoint Higgs field is represented by a general 10×10 antisymmetric matrix with components $\Psi_{i,j}(x^{\mu}, z)$, where $i, j \in \{1, \ldots, 10\}$. The potential is given by the most general 4th order polynomial, in the fields Φ and Ψ, which is fixed by all elements of the above Lie groups.

Physical considerations dictate that they must solve the Euler–Lagrange equations for the Higgs field and the solitary wave consistently; however, they may neglect all other fields in the theory. The system of Euler–Lagrange equations plus boundary conditions for the Higgs field and the solitary wave, the **DWB** equations, are

$$0 = \Box\Phi + \frac{K}{2}\Phi \mathrm{Tr}\Psi^2 + \lambda\Phi\left(\Phi^2 - \nu^2\right),$$

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