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Multiple positive solutions for p-Laplace elliptic equations involving concave–convex nonlinearities and a Hardy-type term

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ABSTRACT

In this paper, the following problem is considered:

$$\begin{cases} -\Delta_p u - \mu \dfrac{|u|^{p-2}u}{|x|^p} = \lambda f(x)|u|^{q-2}u + g(x)|u|^{p^*-2}u, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain such that $0 \in \Omega$, $1 < q < p$, $\lambda > 0$, $\mu < \bar{\mu}$, f and g are nonnegative functions, $\bar{\mu} = (\frac{N-p}{p})^p$ is the best Hardy constant and $p^* = \frac{Np}{N-p}$ is the critical Sobolev exponent. By extracting the Palais–Smale sequence in the Nehari manifold, the existence of multiple positive solutions to this equation is verified.

1. Introduction

In this paper, we are concerned with the following problem:

$$\begin{cases} -\Delta_p u - \mu \dfrac{|u|^{p-2}u}{|x|^p} = \lambda f(x)|u|^{q-2}u + g(x)|u|^{p^*-2}u, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$ $(N \geq p^2)$ is a bounded domain with the smooth boundary $\partial\Omega$ such that $0 \in \Omega$, $\Delta_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian operator, $1 < q < p$, $\lambda > 0$, $\mu < \bar{\mu}$, f and g are nonnegative functions, $\bar{\mu} = (\frac{N-p}{p})^p$ is the best Hardy constant and $p^* = \frac{Np}{N-p}$ is the critical Sobolev exponent.

Let $W_0^{1,p}(\Omega)$ be the completion of $C_0^\infty(\Omega)$ with respect to the norm $(\int_\Omega |\nabla u|^p \,dx)^{1/p}$. The energy functional of (1.1) is defined on $W_0^{1,p}(\Omega)$ by

$$J_\lambda(u) = \frac{1}{p}\int_\Omega \left(|\nabla u|^p - \mu \frac{|u|^p}{|x|^p}\right) dx - \frac{\lambda}{q}\int_\Omega f|u|^q dx - \frac{1}{p^*}\int_\Omega g|u|^{p^*} dx.$$

Then $J_\lambda \in C^1(W_0^{1,p}(\Omega), \mathbb{R})$.$u \in W_0^{1,p}(\Omega) \setminus \{0\}$ is said to be a solution of (1.1) if $\langle J_\lambda'(u), v\rangle = 0$ for all $v \in W_0^{1,p}(\Omega)$ and a solution of (1.1) is a critical point of J_λ.

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