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A variational semilinear singular system

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ABSTRACT

In this paper we study existence of solutions for the following variational semilinear singular system:

$$\begin{cases} u > 0 \quad \text{in } \Omega, \; u \in H_0^1(\Omega) : -\text{div}(A(x)\nabla u) = \theta \dfrac{z^p}{u^{1-\theta}}, \\ z > 0 \quad \text{in } \Omega, \; z \in H_0^1(\Omega) : -\text{div}(M(x)\nabla z) = p u^{\theta} z^{p-1}, \end{cases}$$

where Ω is a bounded open subset of $\mathbb{R}^N$, $N > 2$, A and M are bounded measurable elliptic matrices, and p and θ are such that

$$0 < \theta < 1, \quad p > 0, \quad \theta + p < 2^*.$$

1. Introduction and statement of results

In this paper we study existence of solutions for the system

$$\begin{cases} u > 0 \quad \text{in } \Omega, \; u \in H_0^1(\Omega) : -\text{div}(A(x)\nabla u) = \theta \dfrac{z^p}{u^{1-\theta}}, \\ z > 0 \quad \text{in } \Omega, \; z \in H_0^1(\Omega) : -\text{div}(M(x)\nabla z) = p z^{p-1} u^{\theta}. \end{cases} \tag{1.1}$$

Here Ω is a bounded, open subset of $\mathbb{R}^N$, $N > 2$, p and θ are positive real numbers such that

$$0 < \theta < 1 < p, \tag{1.2}$$

and

$$p + \theta < 2^*, \tag{1.3}$$

and A and M are symmetric and measurable matrices such that

$$\alpha|\xi|^2 \le A(x)\xi\xi \le \beta|\xi|^2, \qquad \alpha|\xi|^2 \le M(x)\xi\xi \le \beta|\xi|^2, \tag{1.4}$$

with $0 < \alpha \le \beta$, for almost every $x \in \Omega$, and for every $\xi \in \mathbb{R}^N$.

The main difficulty in the study of system (1.1) lies in the first equation, where we have u in the denominator of the right-hand side $\frac{z^p}{u^{1-\theta}}$ and, at the same time, the boundary condition $u = 0$.

Our approach is a variational one, so that we define, for v and w in $H_0^1(\Omega)$, the functional

$$J(v, w) = \frac{1}{2}\int_\Omega A(x)\nabla v \nabla v + \frac{1}{2}\int_\Omega M(x)\nabla w \nabla w - \int_\Omega v_+^{\theta} w_+^{p}, \tag{1.5}$$

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