Nonlinear Analysis 74 (2011) 501–515

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

A Browder degree theory from the Nagumo degree on the Hilbert space of elliptic super-regularization

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ARTICLE INFO

Article history:
Received 24 November 2009
Accepted 4 September 2010

MSC:
primary 47H14
secondary 47H05
47H11

Keywords:
Browder degree theory
Nagumo degree theory
Skrypnik degree theory
Maximal monotone operator
Bounded demicontinuous operator of type (S_+)
Compact operator
Elliptic super-regularization

ABSTRACT

Let X be a real reflexive separable locally uniformly convex Banach space with locally uniformly convex dual space X^*. Let $Q : H \to X$ be a linear compact injection, according to Browder and Ton, such that $\overline{Q(H)} = X$, where H is a real separable Hilbert space. A degree mapping d on X is constructed from the Nagumo degree d_{NA} on H by

$$d(T+f, G, 0) := \lim_{t\to 0} d_{NA}\left(I + \frac{1}{t}Q^*(T_t + f)Q, Q^{-1}G, 0\right),$$

where $G \subset X$ is open and bounded, T_t is the resolvent $(T^{-1} + tJ^{-1})^{-1}$ of a strongly quasibounded maximal monotone operator $T : X \supset D(T) \to 2^{X^*}$ with $0 \in T(0)$, and $f : \overline{G} \to X^*$ is demicontinuous, bounded and of type (S_+). A "range of sums" result is also given, using the Skrypnik degree theory, in order to further exhibit the methodology of "elliptic super-regularization".

1. Introduction — preliminaries

Unless otherwise stated, the symbol X stands for a real reflexive locally uniformly convex Banach space with locally uniformly convex dual space X^*. The symbols $\|\cdot\|_X$ and $\|\cdot\|_{X^*}$ stand for the norms of X and X^*, respectively and $J : X \to X^*$ is the normalized duality mapping. The symbol H is reserved for a real separable Hilbert space. In what follows, "continuous" means "strongly continuous" and the symbol "$\to$" ("$\rightharpoonup$") means strong (weak) convergence. Also, "demicontinuous" means strong-to-weak continuous. The symbol $\mathcal{R}(\mathcal{R}_+)$ stands for the set $(-\infty, \infty)$ ($[0, \infty)$) and the symbols ∂D, $\overset{\circ}{D}$, $\overline{D}$, denote the strong boundary, interior and closure of the set D, respectively. We denote by $B_r(0)$ the open ball of X, or X^*, or H with center at zero and radius $r > 0$.

For an operator $T : X \to 2^{X^*}$ we denote by $D(T)$ the effective domain of T, i.e. $D(T) = \{x \in X : Tx \neq \emptyset\}$. We denote by $G(T)$ the graph of T, i.e. $G(T) = \{(x, y) : x \in D(T),\ y \in Tx\}$. An operator $T : X \supset D(T) \to 2^{X^*}$ is called "monotone" if for every $x, y \in D(T)$ and every $u \in Tx,\ v \in Ty$ we have

$$\langle u - v, x - y\rangle \geq 0.$$

A monotone operator T is "maximal monotone" if $G(T)$ is maximal in $X \times X^*$, when $X \times X^*$ is partially ordered by inclusion. In our setting, a monotone operator T is maximal if and only if $R(T + \lambda J) = X^*$ for all $\lambda \in (0, \infty)$. If T is maximal monotone, then the operator $T_t \equiv (T^{-1} + tJ^{-1})^{-1} : X \to X^*$ is bounded, demicontinuous, maximal monotone and such that $T_t x \rightharpoonup T^{\{0\}}x$

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0362-546X/$ – see front matter
doi:10.1016/j.na.2010.09.005