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Existence of some neutral partial differential equations with infinite delay

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ABSTRACT

This study intends to investigate a class of quasi-linear partial neutral functional differential equations with infinite delay. We assume that the linear part generates an analytic compact semigroup and the nonlinear part satisfies certain conditions. A sufficient condition is given to ensure the existence of mild and classical solutions. Finally, an example is given to illustrate our abstract results.

1. Introduction

The problem of functional differential equations with infinite delay has been extensively studied. Most of the previous researches were concerned with the following Cauchy problem.

$$\begin{cases} x'(t) = Ax(t) + F(t, x_t), & t \in [0, T], \\ x_0 = \varphi \in \mathcal{P}, \end{cases} \tag{1.1}$$

where the value of $x(t)$ belongs to Banach space X, A generates a C_0-semigroup on X, $\mathcal{P}$ is a phase space of functions mapping $(-\infty, 0]$ into X, F is a function from $[0, T] \times \mathcal{P}$ into X and for each $x : (-\infty, b] \to X$, $b > 0$ and $t \in [0, b]$, x_t represents the "history" of x at time t and is defined by

$$x_t(\theta) = x(t + \theta) \quad \text{for } \theta \in (-\infty, 0].$$

Recently, in [1–3], Henriquez has used the following variation-of-constant formula

$$x(t) = S(t)\varphi(0) + \int_0^t S(t-s)F(s, x_s)\mathrm{d}s$$

to study the existence of solutions and periodic solutions, regularity, and stability of Eq. (1.1). Later, in [4–6], the authors considered the case that the linear part A is a Hille-Yosida operator and extended the results reported in [1–3]. At the same time, Liang, van Castern and Xiao [7,8] solved Eq. (1.1) by using an operator family of more general type. More recently, the non-autonomous case of Eq. (1.1) was considered in [9]. For more details about development and applications on this issue, we refer the reader to the work in [10] and the references therein.

In this paper, we consider the following ordinary neutral functional differential equation with infinite delay

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}(x(t) + F(t, x_t)) = -Ax(t) + G(t, x_t), & t \in [0, T], \\ x_0 = \varphi \in \mathcal{P}, \end{cases} \tag{1.2}$$

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