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On the global well-posedness for the nonlinear Schrödinger equations with large initial data of infinite L^2 norm

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ABSTRACT

It is shown that the Cauchy problem for the nonlinear Schrödinger equations

$$iu_t + \triangle u \pm |u|^{p-1}u = 0, \qquad x \in \mathbb{R}^d,\ t > 0$$

is globally well-posed for a class of initial data which lie in a L^q space with q near 2 when $1 < p < 1 + 4/d$ and $d \le 4$.

1. Introduction

We are interested in the Cauchy problem

$$iu_t + \triangle u \pm |u|^{p-1}u = 0, \quad x \in \mathbb{R}^d,\ t > 0 \tag{1.1}$$

$$u(x, 0) = u_0(x), \quad x \in \mathbb{R}^d \tag{1.2}$$

where $u = u(t, x) : \mathbb{R}^{1+d} \to \mathbb{C}$, $d \le 4$ and $1 < p < 1 + 4/d$. It is well-known (see [1]) that the Cauchy problem is globally well-posed when the initial data lie in $L^2(\mathbb{R}^d)$. There is an extensive literature on this problem whose initial data lie in the subspaces of $L^2(\mathbb{R}^d)$ (see [2,3]). The present paper is an attempt to solve the Cauchy problem (1.1), (1.2) with the data in a larger class of functions which are not included in $L^2(\mathbb{R}^d)$. In [4] Vargas and Vega investigate the case of $p = 3$ and $d = 1$. Kato [5] studies the solvability of (1.1) with the data not in the L^2 framework. The purpose of this paper is to extend their results to many space dimensions in more general function spaces.

Let us describe the content of the paper. In Section 2 we introduce the function space $Y := Z^{(p,\infty)}(L^{2p})$. Then, following the ideas of Vargas and Vega [4] which originate from [2], we construct a unique global solution of the Cauchy problem (1.1), (1.2), where the initial data u_0 can be decomposed as

$$u_0 = \phi_0^N + \psi_0^N \in L^2(\mathbb{R}^d) + Z^{(p,\infty)}(L^{2p}) \tag{1.3}$$

with

$$\|\phi_0^N\|_{L^2} \sim N^\alpha, \qquad \|\psi_0^N\|_Y \lesssim \frac{1}{N} \tag{1.4}$$

for all $N \ge 1$ and some suitable $\alpha > 0$. In Section 3 we propose several sufficient conditions for u_0 to be decomposed as above. Firstly we show that there exists $\delta(d, p) > 0$ such that $u_0 \in L^2 + Y$ for every $u_0 \in L^{2-\delta'}(\mathbb{R}^d)$, $0 < \delta' < \delta(d, p)$. Then if $q < 2$ is close enough to 2, the initial value problem (1.1), (1.2) with the initial data $u_0 \in L^q$ is globally well-posed. In the

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