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On the singular elliptic systems involving multiple critical Sobolev exponents

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ABSTRACT

In this paper, a singular elliptic system is investigated, which involves multiple critical Sobolev exponents and Hardy-type terms. By using variational methods and analytical techniques, the existence of positive and sign-changing solutions to the system is established.

1. Introduction

In this paper, we study the following elliptic system:

$$\begin{cases} -\Delta u - \mu \dfrac{u}{|x|^2} = |u|^{2^*-2}u + \dfrac{\eta\alpha}{\alpha+\beta}|u|^{\alpha-2}|v|^{\beta}u + a_1 u + a_2 v, \\ -\Delta v - \mu \dfrac{v}{|x|^2} = |v|^{2^*-2}v + \dfrac{\eta\beta}{\alpha+\beta}|u|^{\alpha}|v|^{\beta-2}v + a_2 u + a_3 v, \\ u,\ v \in H_0^1(\Omega), \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$ $(N \geq 3)$ is a bounded domain with the smooth boundary $\partial\Omega, 0 \in \Omega,\ a_i \in \mathbb{R},\ i = 1, 2, 3,\ 0 \leq \eta < +\infty,\ -\infty < \mu < \bar{\mu},\ \alpha, \beta > 1, \alpha + \beta = 2^*,\ 2^* := \frac{2N}{N-2}$ is the critical Sobolev exponent, $\bar{\mu} := \left(\frac{N-2}{2}\right)^2$ is the best Hardy constant and the space $H_0^1(\Omega) =: H$ denotes the completion of $C_0^\infty(\Omega)$ with respect to the norm $\left(\int_\Omega |\nabla \cdot|^2 dx\right)^{1/2}$.

We work in the product space $H \times H$. The corresponding energy functional of the problem (1.1) is defined in $H \times H$ by

$$\begin{aligned} J(u, v) := \frac{1}{2}\int_\Omega \left(|\nabla u|^2 + |\nabla v|^2 - \mu\frac{u^2}{|x|^2} - \mu\frac{v^2}{|x|^2}\right) dx - \frac{\eta}{2^*}\int_\Omega |u|^\alpha |v|^\beta dx \\ - \frac{1}{2^*}\int_\Omega \left(|u|^{2^*} + |v|^{2^*}\right) dx - \frac{1}{2}\int_\Omega \left(a_1 u^2 + 2a_2 uv + a_3 v^2\right) dx. \end{aligned} \tag{1.2}$$

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