Nonlinear Analysis 74 (2011) 2765–2777

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

On certain classes of functional inclusions with causal operators in Banach spaces

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ARTICLE INFO

Article history:
Received 12 November 2009
Accepted 23 December 2010

MSC:
primary 47J05
secondary 34A60
34G25
34K09
45D05
47H04
47H08
47H10

Keywords:
Causal operator
Functional inclusion
Cauchy problem
Functional differential inclusion
Volterra integro-differential inclusion
Continuous dependence
Measure of noncompactness
Fixed point
Topological degree
Multivalued map
Condensing map

ABSTRACT

We introduce the notion of a multivalued causal operator and consider an abstract Cauchy problem in a Banach space for various classes of functional inclusions with causal operators. The methods of the topological degree theory for condensing maps are applied to obtain local and global existence results for this problem and to study the continuous dependence of a solution set on initial data. As application we generalize some existence results for semilinear functional differential inclusions and Volterra integro-differential inclusions with delay.

1. Introduction

At the present time the study of systems governed by differential and functional equations with causal operators attracts much attention. The term causal arises from the engineering and the notion of a causal operator turns out to be a powerful tool for unifying problems in ordinary differential equations, integro-differential equations, functional differential equations with finite or infinite delay, Volterra integral equations, neutral functional equations, etc. (see the monograph [1]). Various problems for functional differential equations with causal operators were considered in recent papers [2–5]. In particular, the existence, uniqueness, and continuous dependence of solutions to Cauchy problem in a Banach space are studied in [2,5].

In the present paper we introduce the notion of a multivalued causal operator and consider an abstract Cauchy problem in a Banach space for various classes of functional inclusions with causal operators. The methods of the topological degree

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0362-546X/$ – see front matter
doi:10.1016/j.na.2010.12.024