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Global small solutions to the critical radial Dirac equation with potential

Federico Cacciafesta

SAPIENZA — Università di Roma, Dipartimento di Matematica, Piazzale A. Moro 2, I-00185 Roma, Italy

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ABSTRACT

We solve globally a radial cubic Dirac equation perturbed with a small potential, with data of small critical norm H^1. The main tool is a new endpoint estimate of the perturbed Dirac flow for a class of radial-type initial data.

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1. Introduction

Consider a zero mass nonlinear Dirac equation

$$iu_t - \mathcal{D}u = F(u), \qquad u(0,x) = f(x) \tag{1.1}$$

for the spinor field $u : \mathbb{R}_t \times \mathbb{R}^3_x \to \mathbb{C}^4$, where $\mathcal{D}$ is the operator defined by

$$\mathcal{D} = i^{-1}\sum_{k=1}^{3} \alpha_k \partial_k = -i(\alpha \cdot \nabla)$$

while the 4×4 Dirac matrices are defined as

$$\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix}, \quad k = 1, 2, 3 \tag{1.2}$$

in terms of the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{1.3}$$

We shall assume that the nonlinear term $F(u)$ is cubic of a very specific form, namely

$$\text{either} \quad F(u) = \langle \beta u, u\rangle u \quad \text{or} \quad F(u) = \langle u, u\rangle u, \tag{1.4}$$

E-mail addresses: cacciafe@mat.uniroma1.it, caccia7@hotmail.it.

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