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Mathematical analysis to a nonlinear fourth-order partial differential equation[☆]

Bo Liang [*]

School of Science, Dalian Jiaotong University, Dalian 116028, PR China

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ABSTRACT

The paper first study the steady-state thin film type equation

$$\nabla\cdot(u^n|\nabla\Delta u|^{q-2}\nabla\Delta u)-\delta u^m\Delta u=f(x,u)$$

with Navier boundary conditions in multidimensional space. By the truncation method, a fixed point argument and some energy estimates, the existence and asymptotic limit $\delta\to 0$ for the positive weak solutions are given. Second, the parabolic equation $u_t+(u^n|u_{xxx}|^{q-2}u_{xxx})_x-\delta u^m u_{xx}=0$ with a Navier boundary in one-dimensional space is researched. The existence is obtained by applying a semi-discrete method for the time variable and solving the corresponding elliptic problem. The uniqueness is shown for $q=2$ depending on an energy estimate. In addition, the iteration relation of the semi-discrete problem gives an exponential decay result for the time $t\to\infty$. The thin film equation, which is usually used to describe the motion of a very thin layer of viscous in compressible fluids along an inclined plane, is a class of nonlinear fourth-order parabolic equations and the maximum principle does not hold directly. For applying the classic theory of partial differential equation, the paper transforms the fourth-order problem into a second-order elliptic–elliptic system or a second-order parabolic–elliptic system.

1. Introduction

The thin film equation, which is often used to describe the motion of a very thin layer of viscous incompressible fluids along an inclined plane and is derived from a lubrication approximation, has plenty of applications in physics, medicine and biology etc. Formally, it is a class of fourth-order degenerate partial differential equations and the main difficulty is that the maximum principle and the comparison principle do not hold directly. For the history and development of the thin film equation, the readers may refer to the review paper [1].

Bernis and Friedman [2] first proved the existence of weak solutions and introduced some valuable methods and techniques for the one-dimensional thin film equation

$$u_t+(u^n u_{xxx})_x=0$$

where $n>0$ is a constant and the unknown function u represents the film thickness. Lately, Bertozzi and Pugh [3] proved its existence in the distributional sense and gave the long time behavior by using the entropy functional method. In the paper [4], they studied the thin film equation with a second-order diffusion term in the one-dimensional space and

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* Tel.: +86 13591182534; fax: +86 13591182534.
E-mail address: cnliangbo@163.com.

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