On some class of nonlinear parabolic inequalities in Orlicz spaces

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ABSTRACT

The paper deals with the existence of solutions of some unilateral problems in the framework of Orlicz spaces.

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1. Introduction

We deal with boundary value problems

$$
\begin{cases}
u \geq \psi & \text{a.e. in } Q, \\
\dfrac{\partial u}{\partial t} + \mathcal{A}(u) = g(u)M(|\nabla u|) + f & \text{in } Q, \\
u = 0 & \text{on } \partial Q, \\
u(x, 0) = u_0(x) & \text{in } \Omega
\end{cases}
\tag{P}
$$

where

$$\mathcal{A}(u) = -\operatorname{div}(a(., t, u, \nabla u)),$$

$Q = \Omega \times [0, T]$, $T > 0$ and Ω is a bounded domain of $\mathbf{R}^N$, with the segment property. $a : \Omega \times \mathbf{R} \times \mathbf{R}^N \to \mathbf{R}^N$ is a Carathéodory function (that is, measurable with respect to x in Ω for every (t, s, ξ) in $\mathbf{R} \times \mathbf{R} \times \mathbf{R}^N$, and continuous with respect to (s, ξ) in $\mathbf{R} \times \mathbf{R}^N$ for almost every x in Ω) such that for all $\xi, \xi^* \in \mathbf{R}^N, \xi \neq \xi^*$,

$$a(x, t, s, \xi)\xi \geq \alpha M(|\xi|) \tag{1.1}$$

$$[a(x, t, s, \xi) - a(x, t, s, \xi^*)][\xi - \xi^*] > 0, \tag{1.2}$$

$$|a(x, t, s, \xi)| \leq c(x, t) + k_1 \overline{P}^{-1} M(k_2|s|) + k_3 \overline{M}^{-1} M(k_4|\xi|), \tag{1.3}$$

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