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Nonexistence results for classes of 3×3 elliptic systems

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ABSTRACT

We consider the system

$$\begin{aligned} -\Delta u &= \lambda f(v, w); \quad x \in \Omega \\ -\Delta v &= \mu g(u, w); \quad x \in \Omega \\ -\Delta w &= \sigma h(u, v); \quad x \in \Omega \\ u &= v = w = 0; \quad x \in \partial\Omega, \end{aligned}$$

where Ω is a ball in R^N, $N > 1$ and $\partial\Omega$ is its boundary, λ, μ, σ are positive parameters bounded away from zero, and f, g, h are smooth functions that are negative at the origin (semipositone system) and satisfy certain linear growth conditions at infinity. We establish nonexistence of positive solutions when two of the parameters λ, μ, σ are large. Our proofs depend on energy analysis and comparison methods.

1. Introduction

Consider the system

$$\begin{aligned} -\Delta u &= \lambda f(v, w); \quad x \in \Omega \\ -\Delta v &= \mu g(u, w); \quad x \in \Omega \\ -\Delta w &= \sigma h(u, v); \quad x \in \Omega \\ u &= v = w = 0; \quad x \in \partial\Omega, \end{aligned} \tag{1.1}$$

where Ω is a ball in R^N, $\partial\Omega$ is its boundary, $\lambda, \mu, \sigma \geq \epsilon_0$, where $\epsilon_0 > 0$, and f, g, h satisfy:

(H1) $f, g, h : [0, \infty) \times [0, \infty) \to R$ are C^1 functions such that $f(0, 0) < 0$, $g(0, 0) < 0$, $h(0, 0) < 0$, and $f_v > 0$, $f_w > 0$, $g_u > 0$, $g_w > 0$, $h_u > 0$, $h_v > 0$ for all $u > 0$, $v > 0$, $w > 0$.

(H2) There exist positive constants K_i and M_i, $i = 1, 2, 3$ such that $f(v, w) \geq K_1 v - M_1$, $g(u, w) \geq K_2 w - M_2$ and $h(u, v) \geq K_3 u - M_3$ for all $u > 0$, $v > 0$, $w > 0$.

(H3) There exist positive numbers $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2$ such that $f(\beta_1, 0) = f(0, \gamma_1) = g(\alpha_1, 0) = g(0, \gamma_2) = h(\alpha_2, 0) = h(0, \beta_2) = 0$.

Then we establish:

Theorem 1.1. *Let* (H1), (H2) *and* (H3) *hold. Then the system* (1.1) *has no positive solution if two of the parameters* λ, μ, σ *are large.*

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