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The second expansion of large solutions for semilinear elliptic equations ☆

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ABSTRACT

In this paper, we analyze the influence of the domain geometry in the second expansion of solutions to the boundary blow-up elliptic problem $\triangle u = b(x)f(u)$, $u > 0$, $x \in \Omega$, $u|_{\partial\Omega} = \infty$, where Ω is a bounded domain with C^4-smooth boundary in $\mathbb{R}^N$, $b \in C^\alpha(\bar{\Omega})$ which is positive in Ω and may be vanishing on the boundary, and f is *normalized* regularly varying at infinity with index p $(p > 1)$.

1. Introduction and the main results

In this paper, we consider the second expansion of solutions to the following boundary blow-up elliptic problem

$$\Delta u = b(x)f(u), \quad u > 0,\ x \in \Omega,\ u|_{\partial\Omega} = \infty, \tag{1.1}$$

where the last condition means that $u(x) \to \infty$ as $d(x) = \mathrm{dist}(x, \partial\Omega) \to 0$, Ω is a bounded domain with C^4-smooth boundary in $\mathbb{R}^N$, b satisfies

(b$_1$) $b \in C^\alpha(\bar{\Omega})$ for some $\alpha \in (0, 1)$, is positive in Ω;
(b$_2$) there exist $k \in \Lambda$ and $B_0 \in \mathbb{R}$ such that

$$b(x) = k^2(d(x))\big(1 + B_0 d(x) + o(d(x))\big) \quad \text{near } \partial\Omega,$$

where Λ denotes the set of all positive non-decreasing functions in $C^2(0, \delta_0)$ which satisfy

$$\begin{cases} \lim\limits_{t\to 0^+} \dfrac{K(t)}{k(t)} = 0, \quad K(t) = \displaystyle\int_0^t k(s)\mathrm{d}s; \\ \lim\limits_{t\to 0^+} \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{K(t)}{k(t)}\right) := D_k \in (0, 1]; \\ \lim\limits_{t\to 0^+} t^{-1}\left(\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{K(t)}{k(t)}\right) - D_k\right) := E_k \in \mathbb{R}, \end{cases}$$

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