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An existence result for a class of p-Laplacian elliptic systems involving homogeneous nonlinearities in R^N ☆

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ABSTRACT

In the present paper, we study the existence of nontrivial solutions for a class of p-Laplacian elliptic systems in R^N. A new existence result for nontrivial solutions is obtained by means of variational methods.

1. Introduction and main result

Consider the following p-Laplacian system:

$$\begin{cases} -\Delta_p u + |u|^{p-2}u = \dfrac{1}{\mu}\dfrac{\partial F(u,v)}{\partial u} + f, & \text{in } R^N, \\ -\Delta_p v + |v|^{p-2}v = \dfrac{1}{\mu}\dfrac{\partial F(u,v)}{\partial v} + g, & \text{in } R^N, \\ u, v \in W^{1,p}(R^N), \end{cases} \tag{1.1}$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ denotes the p-Laplacian operator, $N \geq 3$, $1 < p < N$, $p < \mu < p^* = \frac{pN}{N-p}$, and $W^{1,p}(R^N)$ is the Sobolev space with the norm $\|u\|_{1,p} = \left(\int_{R^N}(|\nabla u|^p + |u|^p)\mathrm{d}x\right)^{\frac{1}{p}}$. $F \in C^1(R \times R, R^+)$ is positively homogeneous of degree μ, that is, $F(tu, tv) = t^\mu F(u, v)$ for all $(u, v) \in R \times R$ and $t > 0$, $R^+ = [0, +\infty)$, $f, g \in W^{-1,p'}(R^N) \setminus \{0\}$, where p' is the conjugate to p and $W^{-1,p'}(R^N)$ is the space dual to $W^{1,p}(R^N)$. Problem (1.1) is posed in the framework of the Sobolev space $E = W^{1,p}(R^N) \times W^{1,p}(R^N)$ with the standard norm

$$\|(u,v)\|_E = \left(\int_{R^N}(|\nabla u|^p + |u|^p)\mathrm{d}x + \int_{R^N}(|\nabla v|^p + |v|^p)\mathrm{d}x\right)^{\frac{1}{p}}.$$

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