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Solutions to a gradient system with resonance at both zero and infinity☆

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ABSTRACT

In this paper, we study the existence and multiplicity of nontrivial solutions for a gradient system with resonance at both zero and infinity via Morse theory.

1. Introduction

This paper is concerned with the existence of solutions to the gradient system

$$\begin{cases} -\Delta u = F_u(x,u,v), & x \in \Omega, \\ -\Delta v = F_v(x,u,v), & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega, \end{cases} \tag{GS}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded open domain with smooth boundary $\partial\Omega$, $N \geqslant 3$, and $F \in C^2(\Omega \times \mathbb{R}^2, \mathbb{R})$ satisfies the subcritical growth condition

(F) there are $C > 0$ and $2 < p < \frac{2N}{N-2} := 2^*$ such that

$$|\nabla F(x,z)| \leqslant C(1 + |z|^{p-1}), \quad \text{for } x \in \Omega, z = (u,v) \in \mathbb{R}^2.$$

Let E be the Hilbert space $H_0^1(\Omega) \times H_0^1(\Omega)$ endowed with the inner product

$$\langle (u,v),(\phi,\psi)\rangle = \int_\Omega (\nabla u \nabla \phi + \nabla v \nabla \psi)\, \mathrm{d}x, \quad (u,v),(\phi,\psi) \in E$$

and associated norm

$$\|z\|^2 = \int_\Omega |\nabla u|^2 + |\nabla v|^2 \mathrm{d}x, \quad z = (u,v) \in E.$$

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