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Global existence of strong solutions of Navier–Stokes equations with non-Newtonian potential for one-dimensional isentropic compressible fluids[☆]

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ABSTRACT

The aim of this paper is to discuss the global existence and uniqueness of strong solution for a class of the isentropic compressible Navier–Stokes equations with non-Newtonian in one-dimensional bounded intervals. We prove two global existence results on strong solutions of the isentropic compressible Navier–Stokes equations. The first result shows only the existence, and the second one shows the existence and uniqueness result based on the first result, but the uniqueness requires some compatibility condition.

1. Introduction

In this paper, we consider a class compressible Navier–Stokes equations with non-Newtonian potential for one-dimensional isentropic compressible fluids

$$\begin{cases} \rho_t + (\rho u)_x = 0 & \text{in } (0,T)\times(0,1) \quad (a) \\ (\rho u)_t + (\rho u^2)_x + \rho\Phi_x - \lambda u_{xx} + P_x = \rho f & \text{in } (0,T)\times(0,1) \quad (b) \\ \left(\left(\dfrac{\delta(\Phi_x)^2+1}{(\Phi_x)^2+\delta}\right)^{\frac{2-p}{2}}\Phi_x\right)_x = 4\pi g\left(\rho - \dfrac{1}{|\Omega|}\displaystyle\int_\Omega \rho \mathrm{d}x\right) & \text{in } (0,T)\times(0,1). \quad (c) \end{cases} \tag{1.1}$$

Here the unknown functions $\rho = \rho(x,t)$ and $u = u(x,t)$ denote the density and velocity, respectively. $P = a\rho^\gamma (a > 0, \gamma > 1)$ is the pressure. $\Phi = \Phi(x,t)$ is the non-Newtonian gravitational potential; $1 < p < 2$, $0 < \delta < 1$. In the rest of this paper, we normalize $a = 1$. Physically, this system describes the motion of compressible viscous isentropic flow under the non-Newtonian gravitational force.

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