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Global regularity for divergence form elliptic equations in Orlicz spaces on quasiconvex domains[☆]

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ABSTRACT

In this paper, we derive the global regularity of weak solutions to divergence form elliptic equations in the Orlicz space $W_0^1 L^\phi$ with $\phi \in \nabla_2 \cap \Delta_2$. The domain considered here is quasiconvex, which is a generalization of the Reifenberg flat domain.

1. Introduction

This paper is concerned with the regularity of weak solutions to

$$\begin{cases} -\mathrm{div}(A\nabla u) = \mathrm{div}\,\mathbf{f}, & x \in \Omega, \\ u = 0, & x \in \partial\Omega. \end{cases} \tag{1.1}$$

Here A is a symmetric matrix which is uniformly elliptic and belongs to BMO [1]. We say $u \in W_0^{1,p}(\Omega)$ $(1 < p < \infty)$ is a weak solution to (1.1), if $\forall \psi(x) \in W_0^{1,p^*}(\Omega)$ with $\frac{1}{p} + \frac{1}{p^*} = 1$,

$$\int_\Omega \nabla u \cdot \nabla \psi \mathrm{d}x + \int_\Omega \mathbf{f}(x) \cdot \nabla \psi(x) \mathrm{d}x = 0.$$

The behavior of solutions to (1.1) is well understood on smooth domains. In the nonsmooth case, for example, if Ω is only Lipschitz, Savaré [2] studied the boundary regularity for general quasilinear elliptic equations by way of a variational method based on Nirenberg difference quotients; Byun and Wang [3] investigated the $W^{1,p}$ global estimates of (1.1) on the Reifenberg flat domain, and they proved that for $1 < p < \infty$, there exists a small δ, such that on all (δ, R)-Reifenberg flat domain Ω,

$$\|\nabla u\|_{L^p(\Omega)} \le C\|\mathbf{f}\|_{L^p(\Omega)}. \tag{1.2}$$

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