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Bifurcation of sign-changing solutions for one-dimensional p-Laplacian with a strong singular weight: p-superlinear at ∞

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ABSTRACT

We investigate Dirichlet boundary value problems of one-dimensional p-Laplace equations with a singular weight which may not be in L^1. Using the properties of eigenfunctions and the global bifurcation theory and considering the case, p-superlinear at ∞, we obtain the similar results as seen in [1] of the case, p-sublinear at ∞. Moreover, we obtain the existence of sign-changing solutions when the nonlinear term is asymptotically p-sublinear near 0 and p-superlinear at ∞.

1. Main results

In this paper, we investigate the following one-dimensional p-Laplacian problem with a weight function

$$\begin{cases} \varphi_p(u'(t))' + \lambda h(t)f(u(t)) = 0, & \text{a.e. in } (0,1), \\ u(0) = u(1) = 0, \end{cases} \tag{P_λ}$$

where $\varphi_p(s) = |s|^{p-2}s$, $p > 1$, λ is a nonnegative parameter, h is a nonnegative measurable function on $(0, 1)$, $h \not\equiv 0$ on any open subinterval in $(0, 1)$ which may be singular at $t = 0$ and/or $t = 1$ and $f \in C(\mathbb{R}, \mathbb{R})$. For the motivation and history, one may refer to [1] and references therein.

In [1], under the assumptions

(F1) $sf(s) > 0$ for $s \neq 0$,
(F2) $0 < f_0 \equiv \lim_{s\to 0^+} f(s)/s^{p-1} < \infty$,
(F3) $f_\infty = \lim_{s\to\infty} f(s)/s^{p-1} = 0$

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