Nonlinear Analysis 74 (2011) 2548–2562

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Global attractor for a system of Klein–Gordon–Schrödinger type in all $\mathbb{R}$

Marilena N. Poulou, Nikolaos M. Stavrakakis *

Department of Mathematics, National Technical University Athens, Zografou Campus 157 80, Athens, Hellas, Greece

ARTICLE INFO

Article history:
Received 15 November 2010
Accepted 5 December 2010

MSC:
35B40
35B45
35B65
35D05
35D10
35J50
35J70
35P30

Keywords:
Klein–Gordon–Schrödinger system
Global attractor
Absorbing set
Continuity
Asymptotic compactness
Unbounded domain

ABSTRACT

In this paper we study the long time behavior of solutions for the following system of Klein–Gordon–Schrödinger type

$$
\begin{aligned}
&i\psi_t + \kappa\psi_{xx} + i\alpha\psi = \phi\psi + f,\\
&\phi_{tt} - \phi_{xx} + \phi + \lambda\phi_t = -\mathrm{Re}\,\psi_x + g,\\
&\psi(x,0) = \psi_0(x), \qquad \phi(x,0) = \phi_0(x), \qquad \phi_t(x,0) = \phi_1(x),\\
&\lim_{x\to\pm\infty}\psi(x,t) = \lim_{x\to\pm\infty}\phi(x,t) = 0, \quad t > 0,
\end{aligned}
$$

where $x \in \mathbb{R}$, $\kappa > 0$, $\alpha > 0$, $\lambda > 0$. First, the existence, uniqueness and continuity of the solutions on the initial data are proved. Then the asymptotic compactness of the solutions and the existence of a global compact attractor are shown.

1. Introduction

The aim of this paper is to prove the existence of a global compact attractor for the following system

$$i\psi_t + \kappa\psi_{xx} + i\alpha\psi = \phi\psi + f, \tag{1.1}$$

$$\phi_{tt} - \phi_{xx} + \phi + \lambda\phi_t = -\mathrm{Re}\,\psi_x + g, \tag{1.2}$$

$$\psi(x,0) = \psi_0(x), \qquad \phi(x,0) = \phi_0(x), \qquad \phi_t(x,0) = \phi_1(x), \tag{1.3}$$

$$\lim_{x\to\pm\infty}\psi(x,t) = \lim_{x\to\pm\infty}\phi(x,t) = 0, \quad t > 0, \tag{1.4}$$

where $x \in \mathbb{R}, t > 0, \kappa > 0, \alpha > 0, \lambda > 0$. Also f, g are complex and real valued functions, respectively. The complex valued variable ψ stands for the dimensionless low frequency electron field, whereas the real valued variable ϕ denotes the dimensionless low frequency density. The system (1.1)–(1.4) describes the nonlinear interaction between high frequency electron waves and low frequency ion plasma waves in a homogeneous magnetic field, adapted to model the UHH plasma heating scheme. The dissipative mechanism of the system is introduced by the terms $i\alpha\psi$ and $\lambda\phi_t$.

Systems of Klein–Gordon–Schrödinger type have been studied for many years. To our knowledge, it seems that the first problem of this type is the so-called *Yukawa System*, which goes back to 1935. Another model which is of the same type is the

* Corresponding author.
E-mail addresses: mpoulou@math.ntua.gr (M.N. Poulou), nikolas@central.ntua.gr (N.M. Stavrakakis).

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doi:10.1016/j.na.2010.12.009