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Periodicity and stability for a Lotka–Volterra type competition system with feedback controls and deviating arguments

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ABSTRACT

This paper deals with the general periodic Lotka–Volterra type competition systems with feedback controls and deviating arguments. By employing fixed point index theory on cone, an explicit necessary and sufficient condition for the global existence of the positive periodic solution of the systems is proved. By constructing a suitable Lyapunov functional, a set of easily verifiable sufficient conditions for the global asymptotic stability of the positive periodic solution of the systems is given.

1. Introduction

Consider the following general nonautonomous Lotka–Volterra n-species competition system (1.1)((1.1$_a$)–(1.1$_b$)) with feedback controls and deviating arguments

$$\begin{aligned}\dot{y}_i(t) &= b_i(y_i(t))\Bigg[r_i(t)-\sum_{j=1}^{n}a_{ij}(t)y_j(t)-\sum_{j=1}^{n}\sum_{k=1}^{m}b_{ijk}(t)y_j(t-\tau_{ijk}(t))\\ &\quad -\sum_{j=1}^{n}\sum_{k=1}^{m}\int_{-\infty}^{t}c_{ijk}(t,\tau)y_j(\tau)\mathrm{d}\tau-d_i(t)u_i(t)-e_i(t)u_i(t-\sigma_i(t))\Bigg], \qquad (1.1_a)\\ \dot{u}_i(t) &= -\alpha_i(t)u_i(t)+\beta_i(t)y_i(t)+\gamma_i(t)y_i(t-\delta_i(t)), \qquad (1.1_b)\end{aligned}$$

where u_i, $1 \le i \le n$, denote indirect feedback control variables [1]. Throughout this paper, we use $i, j = 1, 2, \ldots, n$, $k = 1, 2, \ldots, m$ unless otherwise stated. Let $R = (-\infty, \infty)$, $R_+ = [0, \infty)$. For system (1.1), we introduce the following hypotheses:

(H$_1$) $r_i, \alpha_i \in C(R, R)$, $a_{ij}, b_{ijk}, d_i, e_i, \beta_i, \gamma_i \in C(R, R_+)$ are ω-periodic functions with $\int_0^{\omega} r_i(t)\mathrm{d}t > 0$ and $\int_0^{\omega}\alpha_i(t)\mathrm{d}t > 0$, $\omega > 0$;

(H$_2$) $c_{ijk}(t+\omega, s+\omega) = c_{ijk}(t, s) \ge 0$ and $\int_{-\infty}^{t} c_{ijk}(t,\tau)\mathrm{d}\tau$ is continuous with respect to t and $\int_0^{\infty}\int_{-u}^{0} c_{ijk}(s+u, s)\mathrm{d}s\mathrm{d}u < \infty$;

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