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Generalized KKM theorems and common fixed point theorems

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ABSTRACT

In this paper, we first prove a generalized KKM theorem, and then use this generalized KKM theorem to establish the generalized equi-KKM theorem, common fixed point theorems for a family of multivalued maps, and the Kakutani–Fan–Glicksberg fixed point theorem. We also show that an existence theorem of the common fixed point theorem is equivalent to the Kakutani–Fan–Glicksberg fixed point theorem.

1. Introduction

In 1929, Knaster et al. [1] first established the famous KKM theorem in finite dimensional spaces. In 1961, Fan [2] extended the KKM theorem to infinite dimensional topological vector spaces and gave applications in several directions. See, for example [3], for a full account of the development. Many authors have made important contributions to the development of the KKM principle and its applications. For example, one can see [4–11] and references therein.

Before we state the results which are related to this paper, we introduce some notations and definitions. For details, one can see [2,5,10,12,13].

Definition 1.1 (*[2]*). Let X be a nonempty subset of a topological vector space E. Then a multivalued map $T : X \multimap E$ is called KKM map if $\mathrm{co}\{x_1, x_2, \dots, x_n\} \subseteq \cup_{i=1}^{n} T(x_i)$ for each finite subset $\{x_1, x_2, \dots, x_n\}$ of X.

Theorem 1.1 (*[2]*). *Let X be a nonempty subset of a Hausdorff topological vector space E, and $T : X \multimap E$ be a KKM map with nonempty closed values. If $T(x_0)$ is a compact set for some $x_0 \in X$, then $\cap_{x\in X} T(x) \neq \emptyset$.*

In 1991, Chang and Zhang [5] studied generalized KKM theorems in the Hausdorff topological vector space.

Definition 1.2 (*[5]*). Let Y be an arbitrary nonempty set. Let X be a nonempty subset of a vector space E. A multivalued map $T : Y \multimap X$ is said to be a generalized KKM map if for any nonempty finite subset $\{y_1, \dots, y_n\}$ of Y, there exists a finite set $\{x_1, \dots, x_n\}$ in X with $\mathrm{co}(\{x_1, \dots, x_n\}) \subseteq X$ such that $\mathrm{co}\{x_i : i \in I\} \subseteq \cup_{i\in I} T(y_i)$ for each nonempty subset I of $\{1, 2, \dots, n\}$.

Theorem 1.2 (*[5]*). *Let E be a Hausdorff topological vector space, and X a nonempty convex subset in E. Suppose $T : X \multimap E$ is a generalized KKM map with nonempty closed values, and there exists $x_0 \in X$ such that $T(x_0)$ is a compact subset of E. Then $\cap_{x\in X} T(x) \neq \emptyset$.*

In 2000, Kirk et al. [10] established the KKM principle in hyperconvex metric spaces and obtained some of its applications.

Let A be a bounded subset of a metric space (M, d), and $\mathrm{ad}(A) := \cap\{B \subseteq M : B$ is a closed ball in M such that $A \subseteq B\}$.

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