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Positive solutions of semipositone higher-order differential equations on time scales☆

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ABSTRACT

In this paper, we are concerned with the following $2n$th-order differential equations on time scales

$$\begin{cases}(-1)^n y^{\Delta^{2n}}(t) = g(t)f(t, y(t)), \quad t \in [a, b],\\ y^{\Delta^{2i}}(a) - \beta_{i+1} y^{\Delta^{2i+1}}(a) = \alpha_{i+1} y^{\Delta^{2i}}(\nu),\\ \gamma_{i+1} y^{\Delta^{2i}}(\nu) = y^{\Delta^{2i}}(b), \quad 0 \le i \le n-1,\end{cases}$$

where $\nu \in (a, b)$, $n \ge 1$, $\beta_i > 0$, $1 < \gamma_i < \frac{b-a+\beta_i}{\nu-a+\beta_i}$, $0 \le \alpha_i < \frac{b-\gamma_i\nu+(\gamma_i-1)(a-\beta_i)}{b-\nu}$, $i = 1, 2, \ldots, n$. The functions $g : [a, b] \to [0, +\infty)$ and $f : [a, b] \times [0, +\infty) \to (-\infty, +\infty)$ are continuous, or g is singular at $t = a$ and/or $t = b$. We obtain some properties and sharp estimates of the corresponding Green's function and investigate the existence of positive solutions of the semipositone problems for $2n$-order differential equations by the use of the property of Green's function, variable transformation and the fixed point index theorem.

1. Introduction

In this paper, we are concerned with the following $2n$th-order differential equations on time scales

$$\begin{cases}(-1)^n y^{\Delta^{2n}}(t) = g(t)f(t, y(t)), \quad t \in [a, b],\\ y^{\Delta^{2i}}(a) - \beta_{i+1} y^{\Delta^{2i+1}}(a) = \alpha_{i+1} y^{\Delta^{2i}}(\nu),\\ \gamma_{i+1} y^{\Delta^{2i}}(\nu) = y^{\Delta^{2i}}(b), \quad 0 \le i \le n-1,\end{cases} \tag{1.1}$$

where $\nu \in (a, b)$, $n \ge 1$, $\beta_i > 0$, $1 < \gamma_i < \frac{b-a+\beta_i}{\nu-a+\beta_i}$, $0 \le \alpha_i < \frac{b-\gamma_i\nu+(\gamma_i-1)(a-\beta_i)}{b-\nu}$, $i = 1, 2, \ldots, n$; The functions $g : [a, b] \to [0, +\infty)$ and $f : [a, b] \times [0, +\infty) \to \mathbb{R}$ are continuous and there exists a constant $M > 0$ such that $f(t, y) \ge -M$ for all $t \in [a, b]$ and $y \ge 0$. Meanwhile, we also consider the case where the function g may be singular at $t = a$ and/or $t = b$.

Two-point and multi-point boundary value problems for second-order and higher-order differential equations play an important role in both theory and application, and as a consequence, have attracted many researcher's attention in recent years (see [1–18] and the references therein). They are often used to model various phenomena in physics, chemistry, biology

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