



Low-regularity solutions of the periodic general Degasperis–Procesi equation

Lixin Tian^{a,*}, Yuexia Chen^a, Yue Liu^b, Ying Gao^a

^a Nonlinear Scientific Research Center, Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu, 212013, PR China

^b Department of Mathematics, University of Texas, Arlington, TX 76019, USA

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ABSTRACT

This paper studies low-regularity solutions of the periodic general Degasperis–Procesi equation with an initial value. The existence and the uniqueness of solutions are proved. The results are illustrated by considering the periodic peakons of the periodic general Degasperis–Procesi equation.

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1. Introduction

In [1–3], Degasperis and Procesi first studied the following family of third-order dispersive PDE conservation laws:

$$u_t + c_0 u_x + \gamma u_{xxx} - \alpha^2 u_{txx} = (c_1 u^2 + c_2 u_x^2 + c_3 uu_{xx})_x \quad (1.1)$$

where $\alpha, c_i, i = 0, 1, 2, 3$, are real constants. When $c_1 = -\alpha/2$, $c_2 = \frac{\varepsilon(\beta-1)}{2}$ and $c_3 = \varepsilon$, replacing c_0 with k and α^2 with ε in the equation above, we obtain the following equation:

$$\begin{cases} (u - \varepsilon u_{xx})_t + ku_x + \alpha uu_x + \gamma u_{xxx} = \varepsilon(\beta u_x u_{xx} + uu_{xxx}), & x \in \mathbb{R}, t > 0 \\ u(x, 0) = u_0(x), \end{cases} \quad (1.2)$$

where $u(x, t)$ is the fluid velocity at time t in the spatial x direction (or equivalently the height of the free surface of water above a float bottom), k is a constant related to the critical shallow water wave speed, and $\alpha, \beta, \varepsilon$ are dispersion parameters. It is necessary to point out that Eq. (1.2) is equivalent to Eq. (1.1) since when $\varepsilon = \alpha^2 = c_3, k = c_0, \alpha = -2c_1$, and, $\beta = 1 + \frac{2c_2}{\varepsilon}$, Eq. (1.2) turns out to be Eq. (1.1). To better understand the common properties of Eq. (1.1), we resort to studying Eq. (1.2), which is convenient for us to research.

There are at least three famous equations that satisfy the complete integrability condition within this family: the KdV equation (see [4]), the Camassa–Holm equation (see [5,6]), and the Degasperis–Procesi equation (see [1–3]). Besides, Eq. (1.2) includes the family of equations (see [7–9]) and the Fornberg–Whitham equation (see [10]) as special cases, but they are not completely integrable.

With $\varepsilon = 0$ in Eq. (1.2), it becomes the well-known Korteweg–de Vries equation, which describes the unidirectional propagation of waves at the free surface of shallow water under the influence of gravity. $\varepsilon = 0$ represents the wave height

* Corresponding author.

E-mail addresses: tianlx@ujs.edu.cn (L. Tian), yliu@uta.edu (Y. Liu).