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A fixed point theorem for weakly Zamfirescu mappings

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ABSTRACT

In [5], Zamfirescu (1972) gave a fixed point theorem that generalizes the classical fixed point theorems by Banach, Kannan, and Chatterjea. In this paper, we follow the ideas of Dugundji and Granas to extend Zamfirescu's fixed point theorem to the class of weakly Zamfirescu maps. A continuation method for this class of maps is also given.

1. Introduction

Throughout this paper, (X, d) will be a metric space, D a subset of X, and $f : D \to X$ will be a map. We say that f is contractive if there exists $\alpha \in [0, 1)$ such that, for all $x, y \in D$,

$$d(f(x), f(y)) \leq \alpha d(x, y). \tag{C}$$

The well-known Banach's fixed point theorem asserts that, if $D = X$, f is contractive, and (X, d) is complete, then f has a unique fixed point $x \in X$, and for any $x_0 \in X$ the sequence $\{T^n(x_0)\}$ converges to x. This result has been extended by several authors to some classes of maps which do not satisfy the contractive condition (C). For instance, two conditions that can replace (C) in Banach's theorem are the following:

[1] There exists $\alpha \in [0, 1)$ such that, for all $x, y \in X$,

$$d(f(x), f(y)) \leq \frac{\alpha}{2}[d(x, f(x)) + d(y, f(y))]. \tag{K}$$

[2] There exists $\alpha \in [0, 1)$ such that, for all $x, y \in X$,

$$d(f(x), f(y)) \leq \frac{\alpha}{2}[d(x, f(y)) + d(y, f(x))]. \tag{Ch}$$

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