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Existence of solutions for a class of singular quasilinear elliptic resonance problems☆

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ABSTRACT

The paper concerns a resonance problem for a class of singular quasilinear elliptic equations in weighted Sobolev spaces. The equation set studied is one of the most useful sets of Navier–Stokes equations; these describe the motion of viscous fluid substances such as liquids, gases and so on. By using Galerkin-type techniques, the Brouwer fixed point theorem, and a new weighted compact Sobolev-type embedding theorem established by Shapiro, we show the existence of a nontrivial solution.

1. Introduction

In recent years, the existence of solutions for quasilinear elliptic resonance problems has been extensively studied, and many fruitful results have been obtained. One can refer to [1–7]. For example, in [6], Berestycki and Figueiredo considered a weak solution to the following Dirichlet problem in $W_0^{1,2}(\Omega)$:

$$\begin{cases} -\Delta u = \lambda_1 u - au^- + g(u) + f(x), & x \in \Omega, \\ u = 0, & \text{on } \partial\Omega. \end{cases} \tag{P$_1$}$$

Also Rumbos in [7] discussed the existence of a nontrivial solution for a more complicated quasilinear elliptic equation in Sobolev space $H^1_{p,\rho}(\Omega, \Gamma)$:

$$\begin{cases} \mathcal{Q}u = \lambda_1 u\rho - a(x,u)u^-\rho + g(x,u)\rho + h, & x \in \Omega, \\ u = 0, & \text{on } \partial\Omega \end{cases} \tag{P$_2$}$$

where

$$\mathcal{Q}u = -\sum_{i,j=1}^{N} D_i\left[p_i^{\frac{1}{2}}(x)p_j^{\frac{1}{2}}(x)b_{ij}(x)D_j u\right] + \rho c u.$$

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