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Pullback attractors for a non-autonomous generalized 2D parabolic system in an unbounded domain

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ABSTRACT

The existence of a pullback attractor is proven for a non-autonomous generalized 2D parabolic system in an unbounded domain. The asymptotic compactness of the solution operator is obtained by the uniform estimates on the tails of solutions.

1. Introduction

In this paper, we study the dynamical behavior of a non-autonomous generalized 2D parabolic system in an unbounded domain. Let Ω_0 be a bounded open subset in $\mathbb{R}$, $\Omega = \Omega_0 \times \mathbb{R}$ with boundary $\partial\Omega$. Consider a non-autonomous generalized 2D parabolic system

$$\begin{aligned} &-\Delta u_t + \alpha^2 \Delta^2 u_t + \nu \Delta^2 u + \nabla \cdot \overrightarrow{F}(u) + B(u,u) = g(x,t) \quad \text{in } \Omega \times [\tau, \infty), \\ &u = \nabla u = 0 \quad \text{on } \partial\Omega \times [\tau, \infty), \\ &u(x,\tau) = u_\tau(x) \quad \text{in } \Omega, \end{aligned} \tag{1.1}$$

where $u_t = \frac{\partial u}{\partial t}$, α, ν are positive constants, $\overrightarrow{F}$ is a nonlinear vector function, g is an external forcing term with $g \in L^2_{loc}(\mathbb{R}, L^2(\Omega))$ and $B(u,u) = \frac{\partial u}{\partial x_2}\frac{\partial \Delta u}{\partial x_1} - \frac{\partial u}{\partial x_1}\frac{\partial \Delta u}{\partial x_2}$. If $\overrightarrow{F} \equiv 0$ in (1.1), the system is 2D Navier–Stokes–Voight equation. Navier–Stokes-Voight equation was introduced by Oskolkov [1] to describe a Kelvin–Voight viscoelastic incompressible fluid. Many authors have treated the autonomous Navier–Stokes–Voight equation in bounded domains [2,3] and in unbounded domains [4] from various points of view. When the domain is unbounded, the Sobolev embedding is no longer compact. This gives a difficulty for proving the existence of a global attractor. For some PDEs, such difficulty can be overcome by the energy approach, which is introduced by Ball [5,6]. Polat [7] established the existence of a global attractor to the autonomous problem (1.1), that is, when g is independent of time t, in the unbounded domain by using the technique

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