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Multiple solutions of generalized asymptotical linear Hamiltonian systems satisfying Sturm–Liouville boundary conditions[☆]

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ABSTRACT

This paper consider the multiple solutions for even Hamiltonian systems satisfying Sturm–Liouville boundary conditions. The gradient of Hamiltonian function is generalized asymptotically linear. The solutions obtained are shown to coincide with the critical points of a dual functional. Thanks to the index theory for linear Hamiltonian systems by Dong (2010) [1], we find critical points of this dual functional by verifying the assumptions of a lemma about multiple critical points given by Chang (1993) [2].

1. Introduction and main results

In [1], the author established an index theory for the following Hamiltonian system with Sturm–Liouville boundary value conditions

$$\dot{x} = JB(t)x, \tag{1.1}$$

$$x_1(0)\cos\alpha + x_2(0)\sin\alpha = 0, \tag{1.2}$$

$$x_1(1)\cos\beta + x_2(1)\sin\beta = 0, \tag{1.3}$$

where $B \in L^\infty((0,1); GL_s(\mathbf{R}^{2n}))$, $0 \le \alpha \le \pi$ and $0 < \beta \le \pi$, $x = (x_1, x_2) \in \mathbf{R}^n \times \mathbf{R}^n$. That is, for any $B \in L^\infty((0,1); GL_s(\mathbf{R}^{2n}))$, he associated it with a pair of numbers $(i_{\alpha,\beta}(B), \nu_{\alpha,\beta}(B))$, which are called the index and nullity of B respectively. Precisely, he defined $\nu_A(B)$ as the dimension of solution subspace of (1.1)–(1.3), $i_{\alpha,\beta}(B) = i_{\alpha,\beta}(B_0) + I_{\alpha,\beta}(B_0, B)$, where the initial index $i_{\alpha,\beta}(B_0)$ is a prescribed integer associated with B_0, $I_{\alpha,\beta}(B_1, B_2) = \sum_{\lambda\in[0,1)} \nu_{\alpha,\beta}((1-\lambda)B_1 + \lambda B_2)$ when $B_1, B_2 \in L_s(X)$, $B_1 < B_2$. We will introduce the definitions and properties of $i_{\alpha,\beta}(B)$ and $\nu_{\alpha,\beta}(B)$ in detail in Section 2.

In this paper we investigate the following problems (1.2), (1.3) and

$$\dot{x} = JH'(t,x), \tag{1.4}$$

where H' denotes the gradient of H with respect to x. The main result is the following theorem:

Theorem 1.1. *Assume that H, H', H'' are all continuous, that $H(t,0) \equiv 0$, and that the following conditions are satisfied:*

(i) *$H'(t,x) = B_0(t,x)x + o(|x|)$ with $|x| \to 0$, where $B_0(t,x)$ is a symmetric $2n \times 2n$ matrix varying continuously with $(t,x) \in [0,1] \times \mathbf{R}^{2n}$ and satisfying $A_1(t) \le B_0(t,x) \le A_2(t)$, for all (t,x), where A_1 and A_2 are such that $i_{\alpha,\beta}(A_1) = i_{\alpha,\beta}(A_2)$ and $\nu_{\alpha,\beta}(A_2) = 0$.*

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