Nonlinear Analysis 74 (2011) 6770–6780

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Weighted pseudo almost periodic solutions of second order neutral differential equations with piecewise constant argument☆

Li-Li Zhang, Hong-Xu Li*

Department of Mathematics, Sichuan University, Chengdu, Sichuan 610064, PR China

ARTICLE INFO

Article history:
Received 23 December 2010
Accepted 2 July 2011
Communicated by Ravi Agarwal

MSC:
34K14

Keywords:
Neutral differential equation
Weighted pseudo almost periodic sequence
Weighted pseudo almost periodic solution
Piecewise constant argument

ABSTRACT

In this work, we establish a new existence and uniqueness theorem of weighted pseudo almost periodic solution for second order neutral differential equations with piecewise constant argument of the form

$$\frac{d^2}{dt^2}(x(t)+px(t-1))=qx\left(2\left[\frac{t+1}{2}\right]\right)+f(t),$$

where $[\cdot]$ denotes the greatest integer function, p, q are nonzero constants with $|p|\neq 1$, and $f(t)$ is discontinuous weighted pseudo almost periodic. Comparing with the known results, the condition of our result is given explicitly in terms of p, q, which seems simpler and easier to check even in the special case of pseudo almost periodicity.

1. Introduction

Almost periodicity is an attractive topic in the qualitative theory of differential equations due to their significance and applications in physics, mathematical biology, control theory and others. Diagana [1] introduced the weighted pseudo almost periodic functions, which is a natural generalization of the classical pseudo almost periodic functions (see [2,3]), and has been used in the investigation of ordinary differential equations, partial differential equations and functional differential equations (see [4–6]).

The differential equations with piecewise constant argument describe hybrid dynamical systems (a combination of continuous and discrete). These equations have the structure of continuous dynamical systems within intervals and the solution is continuous, and so combine properties of both differential and difference equations. They have applications in certain biomedical models and are similar in structure to those found in certain sequential continuous models of disease dynamics as treated by Busenberg and Cooke (see [7]). Therefore, there are many papers concerning the differential equations with piecewise constant argument (see [8–15] and the references therein).

In this paper, we consider the equation:

$$\frac{d^2}{dt^2}(x(t)+px(t-1))=qx\left(2\left[\frac{t+1}{2}\right]\right)+f(t), \tag{1.1}$$

where p and q are nonzero constants with $|p|\neq 1, f:\mathbb{R}\to\mathbb{R}$, and $[\cdot]$ denotes the greatest integer function. If $f(t)=0$, Papaschinopoulos and Schinas studied the asymptotic behavior of solutions of (1.1) in [16]. For $|p|\neq 1$, (1.1) was studied in [11,8] and some existence and uniqueness results of almost periodic or pseudo almost periodic solution were obtained.

☆ This work is supported by a Grant of NNSF of China (No. 11071042).
* Corresponding author. Tel.: +86 13980727547.
E-mail addresses: zllyou0@gmail.com (L.-L. Zhang), hoxuli@sohu.com (H.-X. Li).

0362-546X/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2011.07.002