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Multiple existence of solutions for a nonhomogeneous elliptic problem on $\mathbb{R}^N$

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ABSTRACT

Let $N \geq 3$, $2^* = 2N/(N-2)$ and $p \in (2, 2^*)$. Our purpose in this paper is to consider the multiple existence of solutions of problem

$$-\varepsilon^2 \Delta u + u = |u|^{p-2}u + \lambda f \quad u \in H^1(\mathbb{R}^N),$$

where $\varepsilon, \lambda > 0, f \in L^2(\mathbb{R}^N) \cap C^2(\mathbb{R}^N), f \geq 0$ and $f \not\equiv 0$.

We will show the effect of the shape of f to the number of solutions.

1. Introduction

Let $N \geq 3$ and $p \in (2, 2^*)$, where $2^* = 2N/(N-2)$. We consider the multiple existence of positive solutions of problem

$$\begin{cases} -\varepsilon^2 \Delta u + u = u^{p-1} + \lambda f & \text{on } \mathbb{R}^N \\ u \in H^1(\mathbb{R}^N) \end{cases} \tag{P}$$

where $f \in C^2(\mathbb{R}^N) \cap L^2(\mathbb{R}^N)$ with $f \geq 0$ and $f \not\equiv 0$. Problem (P) is a variational problem. That is each solution $u \in H^1(\mathbb{R}^N)$ of (P) is a critical point of the functional defined by

$$I(v) = \int_{\mathbb{R}^N} \left(\frac{1}{2}(\varepsilon^2 |\nabla v|^2 + |v|^2) - \frac{1}{p}|v|^p - \lambda f v \right) dx \quad \text{for } v \in H^1(\mathbb{R}^N).$$

In the case that the domain is bounded, the problem (P) has been investigated many authors. Especially the existence of infinitely many solutions of (P) was established in [1] under a restriction of the range of p. In the case that the domain is the entire space $\mathbb{R}^N$, Zhu [2] proved the existence of two positive solutions $u_1, u_2 \in H^1(\mathbb{R}^N)$ of problem (P) for f sufficiently small and

$$0 \leq f(x) \leq Ce^{-(1+\varepsilon)|x|} \quad \text{on } \mathbb{R}^N \text{ for some } \varepsilon, C > 0.$$

The first solution u_1 is close to 0 and the critical value of the second solution u_2 is close to the least energy level c of the homogeneous problem (2.2). This result was improved in [3,4]. In [3], it was shown, under the assumption $|x|^{N-2}f(x)$ is

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