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Infinitely many homoclinic orbits for Hamiltonian systems with indefinite sign subquadratic potentials[☆]

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ABSTRACT

In this paper, we deal with the existence and multiplicity of homoclinic solutions of the second-order Hamiltonian system

$$\ddot{u}(t) - L(t)u(t) + \nabla W(t, u(t)) = 0,$$

where $L(t)$ and $W(t, x)$ are neither autonomous nor periodic in t. Under the assumption that $W(t, x)$ is indefinite sign and subquadratic as $|x| \to +\infty$ and $L(t)$ is a $N \times N$ real symmetric positive definite matrices for all $t \in \mathbb{R}$, we establish some existence criteria to guarantee that the above system has at least one or infinitely many homoclinic solutions by using the genus properties in critical theory.

1. Introduction

Consider the second-order Hamiltonian system

$$\ddot{u}(t) - L(t)u(t) + \nabla W(t, u(t)) = 0, \tag{1.1}$$

where $t \in \mathbb{R}$, $u \in \mathbb{R}^N$, $L : \mathbb{R} \to \mathbb{R}^{N\times N}$ and $W : \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$. As usual, we say that a solution $u(t)$ of (1.1) is homoclinic (to 0) if $u(t) \to 0$ as $t \to \pm\infty$. In addition, if $u(t) \not\equiv 0$ then $u(t)$ is called a nontrivial homoclinic solution.

In the past ten years, the existence and multiplicity of homoclinic solutions of (1.1) have been intensively studied by many authors. Indeed the existence of homoclinic solutions for Hamiltonian systems and their importance in the study of the behavior of dynamical systems have been recognized from Poincaré [1]. Assuming that $L(t)$ and $W(t, x)$ are independent of t or periodic in t, many authors have studied the existence and multiplicity of homoclinic solutions of (1.1) with the aid of critical point theory and variational methods (see for instance [2–9] and the references therein) and some more general Hamiltonian systems are considered in the recent papers [10–13].

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