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Optimization problems on general classes of rearrangements

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ABSTRACT

This paper is concerned with maximization and minimization problems of the energy integral associated to p-Laplace equations depending on functions that belong to a class of rearrangements. We prove existence and uniqueness results, and present some features of optimal solutions. The radial case is discussed in detail. We also prove a result of uniqueness for a class of p-Laplace equations under non-standard assumptions.

1. Introduction

Let Ω be a bounded smooth domain in $\mathbb{R}^N$. If $E \subset \mathbb{R}^N$ is a measurable set, we denote by $|E|$ its Lebesgue measure. We say that two measurable functions $f(x)$ and $g(x)$ defined in Ω have the same rearrangement if

$$|\{x \in \Omega : f(x) \geq \beta\}| = |\{x \in \Omega : g(x) \geq \beta\}| \quad \forall \beta \in \mathbb{R}.$$

If $g_0(x)$ is a bounded function defined in Ω, we denote by $\mathcal{G} = \mathcal{G}(g_0)$ the class of its rearrangements. We assume $g_0(x) > 0$ in a subset of positive measure, and suppose $g_0 \neq$ constant. Let $\overline{\mathcal{G}}$ be the closure of $\mathcal{G}$ in the weak* topology of $L^\infty(\Omega)$. For $1 < p < \infty$, we set $W = H_0^{1,p}(\Omega)$. For $1 \leq q < p,\ g \in \overline{\mathcal{G}}$ and $w \in W$, we define

$$B(g, w) = \frac{q}{p-q} \int_\Omega \left(\frac{p}{q} g|w|^q - |\nabla w|^p \right) dx. \tag{1}$$

Note that, when $g_0(x)$ is sign changing, we may have $g \in \overline{\mathcal{G}}$ with $g(x) \leq 0$ in Ω. If $g(x) \leq 0$ in Ω, then $B(g, w) \leq 0$ for any $w \in W$. If $g(x) > 0$ in a subset of positive measure, we define

$$W_g^+ = \left\{ w \in W : \int_\Omega g|w|^q dx > 0 \right\}.$$

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