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Local well-posedness and stability of peakons for a generalized Dullin–Gottwald–Holm equation

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ABSTRACT

We establish the local well-posedness for a generalized Dullin–Gottwald–Holm equation by using Kato's theory. Furthermore, the orbital stability of the peaked solitary waves is also proved.

1. Introduction

In this paper we consider the following equation:

$$\begin{cases} m_t + (h(u))_x + \gamma u_{xxx} = \alpha^2 \left(\dfrac{g'(u)}{2} u_x^2 + g(u) u_{xx} \right)_x, & t > 0, x \in \mathbb{R}, \\ u(0, x) = u_0(x), & x \in \mathbb{R}, \end{cases} \tag{1.1}$$

where $m = u - \alpha^2 u_{xx}$ and $h(u), g(u) : \mathbb{R} \to \mathbb{R}$ are given C^3-functions. For $h(u) = 2\omega u + \frac{3}{2}u^2$ and $g(u) = u$, Eq. (1.1) becomes the Dullin–Gottwald–Holm equation [1]:

$$\begin{cases} m_t + 2\omega u_x + 3uu_x + \gamma u_{xxx} = \alpha^2 (2u_x u_{xx} + uu_{xxx}), & t > 0, x \in \mathbb{R}, \\ u(0, x) = u_0(x), & x \in \mathbb{R}, \end{cases} \tag{1.2}$$

where $m = u - \alpha^2 u_{xx}$. In [1], Dullin et al. discussed Eq. (1.2) for a unidirectional water wave with fluid velocity $u(t, x)$, with $x \in \mathbb{R}, t \geq 0$, where α^2 and $\frac{\gamma}{c_0}$ are squares of length scales, and $c_0 = \sqrt{gh}$ (where $c_0 = 2\omega$) is the dispersion relation for irrotational water waves propagating at the free surface of a layer of water of average depth h, over a flat bed [2,3]. Eq. (1.2) was derived by using asymptotic expansions directly in the Hamiltonian for Euler's equations in the shallow water regime and thereby shown to be bi-Hamiltonian and has a Lax pair formulation in [1,4]. Eq. (1.2) combines the linear dispersion of the Korteweg–de Vries equation with the nonlinear and nonlocal dispersion of the Camassa–Holm equation [5–7], yet still preserves integrability via the inverse scattering transform method. Further, using a special shallow water wave asymptotic, it was established that the degree of accuracy for the DGH equation is one order higher than that of the KdV equation (in the KdV equation we have $\gamma \neq 0$ while the Camassa–Holm equation is recovered for $\gamma \to 0$ and $\alpha \to 1$).

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