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# Convergence of an implicit iterative process for asymptotically pseudocontractive nonself-mappings

Xiaolong Qin [a,b], Sun Young Cho [c], Tianze Wang [a], Shin Min Kang [d,*]

[a] School of Mathematics and Information Sciences, North China University of Water Resources and Electric Power, Zhengzhou 450011, China

[b] Department of Mathematics, Hangzhou Normal University, Hangzhou 310036, China

[c] Department of Mathematics, Gyeongsang National University, Jinju 660-701, Republic of Korea

[d] Department of Mathematics and the RINS, Gyeongsang National University, Jinju 660-701, Republic of Korea

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ABSTRACT

In this work, an implicit iterative process is considered for asymptotically pseudocontractive nonself-mappings. Weak and strong convergence theorems for common fixed points of a family of asymptotically pseudocontractive nonself-mappings are established in the framework of Hilbert spaces.



## 1. Introduction and preliminaries

Throughout this work, we always assume that $H$ is a real Hilbert space, whose inner product and norm are denoted by $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$, respectively. Let $C$ be a nonempty closed convex subset of $H$ and $T : C \to C$ be a mapping. In this work, we denote the fixed point set of $T$ by $F(T)$.

Recall that $T$ is said to be nonexpansive if

$$\|Tx - Ty\| \le \|x - y\|, \quad \forall x, y \in C.$$

$T$ is said to be asymptotically nonexpansive if there exists a sequence $\{k_n\} \subset [1, \infty)$ with $k_n \to 1$ as $n \to \infty$ such that

$$\|T^n x - T^n y\| \le k_n \|x - y\|, \quad \forall x, y \in C,\ n \ge 1. \tag{1.1}$$

The class of asymptotically nonexpansive mappings was introduced by Goebel and Kirk [1] in 1972. It is known that if $C$ is a nonempty bounded closed convex subset of a Hilbert space $H$, then every asymptotically nonexpansive self-mapping has

* Corresponding author. Fax: +82 557551917.
*E-mail addresses:* qxlxajh@163.com, ljjhqxl@yahoo.com.cn (X. Qin), ooly61@yahoo.co.kr (S.Y. Cho), wangtz@henu.edu.cn (T. Wang), smkang@gnu.ac.kr (S.M. Kang).

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