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A positive solution branch for nonlinear eigenvalue problems in $\mathbb{R}^N$

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ABSTRACT

In this note, we give sufficient conditions for the existence of a positive solution branch for the problem

$$-\Delta u = \lambda a(x) f(u) \quad \text{in } \mathbb{R}^N,$$
$$u(x) \to 0 \quad \text{as } |x| \to \infty,$$

with sign changing weight a, where $N \geq 3$ and f is a smooth nonlinearity with $f(0) \neq 0$.

1. Introduction

Many problems in mathematical physics, for example, wave phenomena, nonlinear field theory, combustion theory, fluid dynamics etc., lead to a nonlinear eigenvalue problem of the type

$$-\Delta u = \lambda f(u) \quad \text{in } \Omega \subset \mathbb{R}^N$$
$$u(x) = 0 \quad \text{on } \partial\Omega,$$

where the existence of a positive solution is of great importance. Recently, many researchers have been interested in studying the following problems in $\Omega \subseteq \mathbb{R}^N$ with weight a:

$$-\Delta u = \lambda a(x) f(u) \quad \text{in } \Omega,$$
$$u(x) = 0 \quad \text{on } \partial\Omega, \tag{1.1}$$

due to the appearance of these kinds of problems in population genetics.

For the case $f(0) = 0$, there are some works dealing with the existence of bounded positive $\mathcal{C}^2$ solutions to (1.1) in $\mathbb{R}^N$ for λ sufficiently large, when $f(1) = 0$, f is smooth and $f(s) > 0$, $s \in (0, 1)$. For example in [1] Tertikas proved the existence result when a is locally Hölder continuous and negative at infinity. For $N \geq 3$, Brown and Stavrakakis [2] proved the same result for when a is smooth and lies in $L^{\frac{N}{2}}$. The decay of solutions are obtained under additional assumptions on the weight a. Gámez [3] established the existence of solutions, in $\mathcal{D}_0^{1,2}(\mathbb{R}^N) :=$ the completion of $\mathcal{C}_c^{\infty}(\mathbb{R}^N)$ with respect the norm $\left(\int_{\mathbb{R}^N} |\nabla u|^2\right)^{\frac{1}{2}}$, to (1.1) when a is continuous and the positive part $a^+ \in L^{\frac{2N}{N+2}}$ and he relaxed the smoothness of f to Lipschitz continuity and differentiability at 0. He proved the decay under the additional assumption that a is bounded.

For the case $f(0) > 0$, some existence results are available when Ω is bounded and λ is sufficiently small. In [4], the authors established the existence of a nonnegative radial solution to (1.1) in B_1, for sign changing a, when f is positive and

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