Nonlinear Analysis 74 (2011) 4573–4580

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Blow-up problem for semilinear heat equation with absorption and a nonlocal boundary condition☆

Alexander Gladkov[a,*], Mohammed Guedda[b]

[a] Department of Mathematics, Vitebsk State University, Moskovskii pr. 33, 210038 Vitebsk, Belarus

[b] Université de Picardie, LAMFA, CNRS, UMR 6140, 33 rue Saint-Leu, F-80039, Amiens, France

ARTICLE INFO

Article history:
Received 22 January 2011
Accepted 6 April 2011
Communicated by Enzo Mitidieri

Keywords:
Semilinear heat equation
Nonlocal boundary condition
Blow-up

ABSTRACT

In this paper we consider a semilinear parabolic equation $u_t = \Delta u - c(x,t)u^p$ for $(x,t) \in \Omega \times (0,\infty)$ with nonlinear and nonlocal boundary condition $u|_{\partial\Omega\times(0,\infty)} = \int_\Omega k(x,y,t)u^l dy$ and nonnegative initial data where $p > 0$ and $l > 0$. We prove some global existence results. Criteria on this problem which determine whether the solutions blow up in finite time for large or for all nontrivial initial data are also given.

1. Introduction

In this paper we consider the following nonlocal initial boundary value problem:

$$\begin{cases} u_t = \Delta u - c(x,t)u^p & \text{for } x \in \Omega,\ t > 0, \\ u(x,t) = \displaystyle\int_\Omega k(x,y,t)u^l(y,t)\mathrm{d}y & \text{for } x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x) & \text{for } x \in \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ for $n \geq 1$ with smooth boundary $\partial\Omega$, $p > 0$ and $l > 0$. Here $c(x,t)$ is a nonnegative locally Hölder continuous function defined for $x \in \overline{\Omega}$ and $t \geq 0$ and $k(x,y,t)$ is a nonnegative continuous function defined for $x \in \partial\Omega$, $y \in \overline{\Omega}$ and $t \geq 0$. The initial datum $u_0(x)$ is a nonnegative continuous function satisfying the boundary condition at $t = 0$.

Many physical phenomena are formulated as nonlocal mathematical models [1–6]. Initial boundary value problem for diffusion and reaction–diffusion equations with linear boundary condition in the second equation of (1.1) has been analyzed by many authors (see, for example, [7–9] and the references therein). Some papers [10–12] deal with the initial boundary value problems with nonlinear and nonlocal boundary conditions. Recently, the initial boundary value problem for reaction–diffusion equation

$$u_t = \Delta u + c(x,t)u^p \quad \text{for } x \in \Omega,\ t > 0$$

with a nonlocal boundary condition in (1.1) has been investigated in [13]. Global existence of solutions with any initial data has been proved for $\max(p,l) \leq 1$. For the case $\max(p,l) > 1$ global existence and blow-up results depend on the behavior of the coefficients $c(x,t)$ and $k(x,y,t)$ as t tends to infinity.

☆ This work was supported in part by DAI, UPJV Amiens and the Regional Council of Picardie.

* Corresponding author. Tel.: +8 10 375 212 514558.
E-mail addresses: gladkoval@mail.ru (A. Gladkov), Mohamed.guedda@u-picardie.fr (M. Guedda).

0362-546X/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2011.04.027