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Homogeneous Einstein–Randers metrics on spheres[☆]

Hui Wang[a], Libing Huang[b], Shaoqiang Deng[b,*]

[a] College of Science, Nanjing University of Posts and Telecommunications, Nanjing 210003, People's Republic of China
[b] School of Mathematical Sciences and LPMC, Nankai University, Tianjin 300071, People's Republic of China

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ABSTRACT

In this paper, we study the homogeneous Einstein–Randers metrics on spheres. It turns out that we can find out all the homogeneous non-Riemannian Einstein–Randers metrics on spheres. Furthermore, we obtain a complete classification of such metrics under isometries. Using this, we present a large number of homogeneous Einstein–Randers metrics of non-constant flag curvature.

1. Introduction

It is our goal in this article to describe the homogeneous Einstein–Randers metrics on spheres and give the complete classification of them under isometries. Randers metrics were introduced by G. Randers in the context of general relativity, and later named by Ingarden. They are built from

- a Riemannian metric $a := a_{ij}dx^i \otimes dx^j$, and
- a 1-form $b := b_i dx^i$, with equivalent description $b^\sharp := b^i \partial_{x^i}$, both living globally on the smooth n-dimensional manifold M. The Finsler function of a Randers metric has the form $F = \alpha + \beta$, where

$$\alpha(x, y) := \sqrt{a_{ij}(x)y^i y^j}, \qquad \beta(x, y) := b_i(x)y^i. \tag{1.1}$$

The Finsler function of a Randers metric satisfies $F(x, y) = F(x, -y)$ if and only if it is Riemannian.

Let (M, F) be a connected Finsler space, $x \in M, y \in T_x(M) \setminus \{0\}$. The *Ricci scalar* $\mathcal{R}ic(x, y)$ is defined to be the sum of those $n-1$ flag curvatures $K(x, y, e_v)$, where $\{e_v : 1 \le v \le n-1\}$ is any collection of $n-1$ orthonormal transverse edges perpendicular to the flagpole, i.e.,

$$\mathcal{R}ic(x, y) := \sum_{v=1}^{n-1} R_{vv}. \tag{1.2}$$

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* Corresponding author.
E-mail address: dengsq@nankai.edu.cn (S. Deng).

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