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On stably quasimonotone hemivariational inequalities

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ABSTRACT

We establish some existence results for variational–hemivariational inequalities of the Hartman–Stampacchia type involving stably quasimonotone set-valued mappings on bounded, closed and convex subsets in reflexive Banach spaces. We also derive a sufficient condition for the existence and boundedness of solutions.

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1. Introduction

This paper discusses *the hemivariational inequality problem* of the Hartman–Stampacchia type:
(P) Find $u \in K$ and $u^* \in F(u)$, such that

$$\langle u^*, v - u\rangle + \int_{\Omega} j^0(x, \hat{u}(x); \hat{v}(x) - \hat{u}(x))\mathrm{d}x \geq 0, \quad \forall v \in K,$$

where K is a nonempty closed convex subset in a real reflexive Banach space X, $F : K \to 2^{X^*}$ is a set-valued mapping, $T : X \to L^p(\Omega; R^k)$ is a linear and continuous mapping, where $1 < p < \infty$, $j(x, y) : \Omega \times R^k \to R$ is a function and Ω is a bounded open set in R^N. We shall denote $\hat{u} := Tu$, $j^0(x, y; h)$ denotes Clarke's generalized directional derivative of a locally Lipschitz mapping $j(x, \cdot)$ at the point $y \in R^k$ with respect to the direction $h \in R^k$, where $x \in \Omega$.

If $T = 0$, then problem (P) is equivalent to finding $u \in K$ and $u^* \in F(u)$, such that

$$\langle u^*, v - u\rangle \geq 0, \quad \forall v \in K,$$

which is called the generalized variational inequality problem.

Hemivariational inequalities have been introduced by Panagiotopoulos [1,2] as the variational formulation of an important class of unilateral or inequality problems in mechanics. By replacing the subdifferential of a convex function by the generalized gradient (in the sense of Clarke) of a locally Lipschitz functional, hemivariational inequalities appear as a mathematical formulation of variational principles for nonconvex, nonsmooth energy problems. The mathematical theory of hemivariational inequalities, as well as their applications in mechanics, engineering or economics were developed by Panagiotopoulos [3–7] in the case of nonconvex energy functions.

When set K is not bounded and the single-valued mapping F is hemicontinuous and stably pseudomonotone with respect to set $U(J, T)$ defined in (2.5), [8] proved that the coercivity condition (A) implies that the solution set of problem (P) is nonempty. When the set K is bounded and the set-valued mapping F is monotone and lower hemicontinuous on K, [9] proved that the solution set of problem (P) is nonempty.

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