



## New generalizations of basic theorems in the KKM theory

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### ABSTRACT

In the present paper, we obtain a new KKM type theorem for intersectionally closed-valued KKM maps and some useful new basic consequences. Typical examples of them are abstract forms of Fan's matching theorem, Fan's geometric lemma, the Fan–Browder fixed point theorem, maximal element theorems, Fan's minimax inequality, variational inequalities, and others.

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### 1. Introduction

The KKM theory, first called by the author [1], is the study on applications of equivalent formulations or generalizations of the KKM theorem due to Knaster, Kuratowski, and Mazurkiewicz in 1929. The KKM theorem provides the foundations for many of the modern essential results in diverse areas of mathematical sciences.

Some of the basic theorems that were applications of the KKM theorem were first obtained by Fan, Browder, and others for convex subsets of topological vector spaces (not necessarily Hausdorff). Later extensions of the theory were due to Lassonde for convex spaces, Horvath for  $H$ -spaces, Park for  $G$ -convex spaces, and others; see [2] and the references therein.

Recently, the KKM theory has been extended to abstract convex spaces by the author and we obtained a large number of new results in such frame; see [3] and the references therein. In fact, there are very large numbers of equivalent formulations, generalizations, and applications of the KKM theorem.

Note that, in many works on the theory, some authors replaced the closed-valuedness of related KKM maps by more general concepts. We noted several times that such replacements are inessential and useless in many cases. However, more recently, Luc et al. [4] have introduced a meaningful concept of *intersectionally closed-valued* multimaps and applied them to several related problems. This new concept is more general than the quite long-standing one of transfer closed-valuedness.

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